Eliciting Subjective Belief Distributions About Returns to Investment in Early Childhood

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I study how parents perceive the uncertainty of the returns to investment in the child development process and how this uncertainty impacts their actual investment in children. To do so, I develop an elicitation procedure of parental subjective belief distributions about the technology of skill production and their subjective investment costs that is guided by a model of parental investment. I collect data using this procedure and extend existing measurement error correction methods that are necessary with belief estimation. I show that parents hold downward-biased mean beliefs about the returns to investment. Moreover, parents who have higher mean beliefs also hold lower levels of uncertainty about their beliefs. Both mean beliefs and uncertainty correlate with actual investment measures. Finally, I estimate a model of parental investment with reference-dependent preferences and show that even though parents hold low mean beliefs, they have a strong incentive to invest if their child is at risk of experiencing a developmental delay.

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1. Introduction

Research by developmental psychologists, sociologists and economists has shown that the development of cognitive and non-cognitive skills throughout early childhood is one of the most important processes in our lives. It impacts not only education attainment and socio-emotional behavior throughout adolescence, but also has long-term consequences for lifetime earnings.[1] One of the key drivers of this process is parental investments.[2] Therefore, understanding why some parents invest in their children more than others is a fundamental question to investigate policies targeting lifetime inequality. Investment gaps that happen early in the childhood can lead to skill gaps in the future that are difficult to remedy (Cunha, Heckman, and Schennach 2010; Heckman et al. 2010).

To help understand how these investment gaps arise, it is crucial to uncover the determinants of parental investment. Past research has highlighted the role of family resources (Dahl and Lochner 2012; Caucutt et al. 2020) and parental characteristics such as maternal education and cognitive skills (Currie and Moretti 2003; Aizer and Stroud 2010; Arendt, Christensen, and Hjorth-Trolle 2021). Recent papers in economics highlight the role of parental information about the process of child development (Cunha, Elo, and Culhane 2022; Boneva and Rauh 2018; Attanasio, Cunha, and Jervis 2019; Dizon-Ross 2019). While this body of work shows that low-income parents often underestimate the returns on investment, and that these beliefs are correlated with lower investment, it does not consider any uncertainty that parents may face about the returns to investment.

This paper aims to address this gap by developing a methodology to elicit and estimate belief distributions about the returns to investment in early childhood guided by a theoretical model of parental investment. I investigate how belief uncertainty about the technology of skill formation affects parental investment decisions in their children. I find that parents in my sample underestimate the returns to investment, consistent with previous findings, and that they hold low uncertainty about them. Moreover, the

[1]Much of the evidence regarding the importance of early childhood development comes from the short and long-term effects of two programs that took place in the 1970s in the United States, the Perry Preschool Project and the Abecedarian Project, which provided free high quality childcare for low income families. See Campbell et al. (2012, 2002b,a); Garca et al. (2020); Schweinhart et al. (1985); Heckman and Karapakula (2019). Duncan, Ziol-Guest, and Kalil (2010) uses the Panel Study of Income Dynamics to link early childhood poverty to adult outcomes.

[2]The theory that parental involvement is fundamental to child development has been developed since Vigotsky's Zone of Proximal Development in the early 1900s. See Ma et al. (2016); Korfmacher et al. (2008) for meta-analysis studies detailing the empirical evidence.

higher their expected returns to investment, the more certain they are about these beliefs, as measured by the variance of their belief distribution. I then estimate a model of parental investment with reference dependent preferences and subjective beliefs, and show that even though parents hold low mean beliefs, they have a strong incentive to invest if their child is at risk of being at a developmental delay. In counterfactual simulations, increasing uncertainty about returns to investment leads to an increase in investment, driven mostly by parents with low expected returns to investment.

Investing in your child is an uncertain process that requires consistent resource commitment for several years with uncertain outcomes. Understanding how belief uncertainty about the returns to investment impacts investment in early childhood is important for many reasons. First, it remains unclear in which direction belief uncertainty affects investment, if at all. On one hand, the literature on schooling decisions for middle and high school students shows that an increase in perceived earnings risk or unemployment risk causes both parents and children to invest less (Attanasio and Kaufmann 2014; Wiswall and Zafar 2015). On the other hand, models that incorporate learning about the returns to investment show that when individuals are highly uncertain about returns (i.e., when prior beliefs have high variance), they may perceive greater potential to learn and thus may invest more (Mira 2007; Badev and Cunha 2012). This paper brings that discussion into the context of early childhood by exploring the empirical relationship between the mean and uncertainty of beliefs and parental investment.

Second, if uncertainty is an important factor in determining parental investment, it is essential to consider it when designing policies that target parental beliefs. There have been many successful parenting interventions which increase investment that focuses on educating parents about the significance of spending quality time with their children, whether through activities such as reading, playing, or engaging in conversations (see, the Jamaica Home Visiting Program and the Nurse-Family Partnership Program). Nevertheless, recent interventions capable of measuring parental beliefs have had mixed results. Some demonstrate that while parents' subjective mean returns to investment increase, their actual investments do not change, while others see an increase in investment but no change in mean beliefs (Attanasio, Cunha, and Jervis 2019; List, Pernaudet, and Suskind 2021).[3] These mixed results indicate that there may

[3] In particular, List, Pernaudet, and Suskind (2021) designed two interventions, one involving parents watching a brief video on the importance of investments, and the other featuring an intensive home visitation instruction and feedback program. Their findings revealed that while parental beliefs changed in both interventions, only the latter led to increased investment. On the other hand, Attanasio, Cunha,

be other factors at play. One potential explanation is that these interventions may be changing parental uncertainty about the returns to investment.

To study parental beliefs, one cannot rely on existing data since it is not possible to estimate subjective preferences or beliefs from observed data alone (Manski 2004). Therefore, I design a survey to elicit the subjective belief distribution and investment decisions from parents. The survey is guided by a theoretical model of parental investment under subjective beliefs about the technology of skill production. For a sample of parents, I present two sets of experiments where parents are presented with specific hypothetical scenarios about the environment faced by a hypothetical child. I then estimate beliefs, costs, and preferences while correcting for possible measurement error.

In the main survey, parents are presented with a hypothetical scenario of either a baby with normal or poor health and possible investments. Parents are asked for that given scenario what is the age range in months that the baby can perform a specific developmental activity. Given the exogenous variation of variables in the scenarios, this age-range question allows me to obtain a measure of parental belief about the skill a child will have in the future in terms of age in months. This is a key aspect, since skill is a latent construct without a natural scale. By phrasing the questions in terms of months, it is possible to measure a child's skill as the developmental delay (or advance) the parent believes a child would develop given the health and investment scenario. Using this exogenous variation, I develop an estimator for the mean belief and uncertainty about the return to investment.

I collect data through an online panel from a sample of women who i) are aged between 18 and 40, ii) have a child, and iii) the oldest child is no more than 5 years of age. This particular sample should have familiarity with early childhood development.[4] I first document several regularities and patterns predicted by the model. Respondents report that they believe children complete harder activities at higher ages. Moreover, respondents report higher ages under scenarios where the child has poor health and low investment, demonstrating that they understand the tradeoffs in child development. These patterns are consistent with model predictions and provide evidence that respondents are engaging with the survey in a meaningful way.

I find that mothers in this sample have low mean beliefs about the returns to invest-

and Jervis (2019) designs an information and education intervention, but while investments changed after the program, their measured mean beliefs did not change.

[4]Individuals who are too far removed from the parenting experience may not have given much thought to child development and could be more likely to respond randomly or without sufficient reflection

ment, consistent with prior studies (e.g., Cunha, Elo, and Culhane (2013); Boneva and Rauh (2018)). On average, mothers believe that a 10% increase in investment leads to a 1.03% increase in their child's human capital, which is about half the returns when estimating using objective data (see, Attanasio, Cunha, and Jervis 2019). Uncertainty about these returns is also low, with a coefficient of variation around 0.53. Notably, mothers with higher mean beliefs tend to be less uncertain: a one standard deviation increase in mean beliefs is associated with a 38% decrease in uncertainty.

To examine how these beliefs relate to behavior, I regress actual time investments on estimated beliefs. Consistent with prior research, higher mean beliefs are associated with greater investment: a one standard deviation increase in mean beliefs predicts a 20% standard deviation increase in daily investment hours. However, greater uncertainty is negatively correlated with investment—a one standard deviation increase in uncertainty predicts a 13% standard deviation decrease in investment.

The data from the belief elicitation allows me to estimate a model of parental investment in children that takes into account subjective beliefs. I incorporate reference-dependent preferences, in which parental investment depends on how they view their child development relative to specific developmental milestones. Using their own beliefs, parents form comparisons that shape their investment behavior. This framework enables me to study how much parents value avoiding developmental delays. I find that parents strongly value their child skill relative to leisure and household consumption. Moreover, when estimating the preference for large developmental delays, I find that parents have a strong incentive to invest if their child is at risk of being at a developmental delay. Overall, even though parents have low beliefs about the returns to investment, they still value their child skill and have a strong incentive to invest if their child is at risk of being at a developmental delay.

Counterfactual analysis shows that increasing either mean beliefs or uncertainty about the returns to investment increases investment. This appears to contradict the reduced-form findings, where higher uncertainty is associated with lower investment. However, this discrepancy can be explained by the negative correlation between mean beliefs and uncertainty: individuals with higher mean beliefs also tend to exhibit lower uncertainty. As a result, the reduced-form relationship may conflate the effects of the two. The counterfactuals help disentangle these channels by isolating their individual contributions within the structural model.

When breaking down the effect of increasing uncertainty on parental investment, I show that there is substantial heterogeneity in investment changes. Parents that increase their investment due to the increase in uncertainty are the ones that hold very low mean beliefs and therefore do not invest much in their child. This is due to uncertainty implying a higher 'risk' of their child falling below developmental thresholds, which induces an increase in investment.

This paper contributes to different strands of literature. First, it builds on a large body of work that uses elicited subjective expectations to inform economic models and understand decision-making under uncertainty (Manski 1993, 2004; Delavande 2008; Jensen 2010; Zafar 2013; Almås, Attanasio, and Jervis 2023). This paper extends that literature by explicitly eliciting not only parents' expectations but also their subjective uncertainty about the returns to early childhood investment—a dimension that has received limited attention in prior work.

Second, it relates to research on parental beliefs and information in early childhood development (Cunha, Elo, and Culhane 2013; Boneva and Rauh 2018; Attanasio, Cunha, and Jervis 2019; Dizon-Ross 2019; List, Pernaudet, and Suskind 2021). While much of this literature focuses on expectations about educational outcomes or the effects of information interventions, my paper contributes by developing a novel belief elicitation procedure that captures both the level and uncertainty of parental beliefs about the skill formation process. Related work has studied adolescent education decisions (Delavande and Zafar 2018; Patnaik et al. 2022; Wiswall and Zafar 2015; Stinebrickner and Stinebrickner 2014), but this paper focuses on early childhood, where beliefs are arguably less informed and more heterogeneous.

Third, the paper contributes to work on uncertainty, risk preferences, and parental investment decisions (Giustinelli 2016; Attanasio and Kaufmann 2014; Carneiro and Ginja 2016; Tabetando 2019; Sovero 2018; Tanaka and Yamano 2015; Basu and Dimova 2022). Prior studies have focused on risk aversion and credit constraints; in contrast, I study how uncertainty in subjective beliefs—rather than risk preferences per se—affects parental investment. This distinction helps clarify why parents may underinvest even when returns are high.

The remainder of this paper is structured as follows. Section 2 describes the economic model and specifies the technology of skill formation, as well as the concepts of subjective expectation and uncertainty. Section 3 describes the survey instrument, how the collected data identifies model primitives and the estimation method. Section 4 describes the data. Section 5 discusses the results, while Section 6 concludes.

2. Model

Consider a mother who must decide how much to invest in her child within a one-period model.[5] Let y_i denote her income, x_i denote her time investment in her child, and c_i denote her consumption. She faces a budget constraint given by

$$c_i + p_i x_i = y_i, \tag{1}$$

where p_i is the relative price of parental investment. Moreover, she faces a time constraint where time investment x_i is bounded by total available time:

$$x_i + h_{l_i} + h_{w_i} = 16. \tag{2}$$

Thus, the mother has 16 hours per day, assuming she sleeps 8 hours, to allocate between her child (x_i), leisure (h_{l_i}), and work (h_{w_i}). I assume that work is not a choice variable but given exogenously.

Let $\theta_{i,0}$ and $\theta_{i,1}$ denote the stock of human capital of child i at birth, $t = 0$, and at 24 months, $t = 1$. Let x_i denote the investment in human capital made by the mother between birth and 24 months. Finally, let ξ_i denote a shock to the development process unknown to the parents. I assume that the *objective* technology of skill formation is given by a Cobb-Douglas formulation:

$$\ln \theta_{i,1} = \delta_0 + \delta_1 \ln \theta_{i,0} + \delta_2 \ln x_i + \xi_i. \tag{3}$$

Parental preferences depend on household consumption, leisure, and child development at the end of the period, $u_i(c_i, h_{l_i}, \theta_{i,1}; \boldsymbol{\alpha})$, where $\boldsymbol{\alpha}$ denotes the vector of parental preferences. The parents maximize their expected utility conditional on the agent's information set Ω_i at the time of the decision. Parents know their own preferences $\boldsymbol{\alpha}$, their income y_i, the price of investment goods p_i, their work hours h_{w_i}, and their child initial stock of human capital, $\theta_{i,0}$. However, they do not know the parameters of the objective technology function, $\delta_0, \delta_1, \delta_2$. Parents have beliefs about these parameters, so from their point of view they are random variables. Consequently, the human capital of the child at $t = 1$ will also be a random variable and I denote its distribution by $G_i(\cdot)$. Therefore, the information set of the parents is the set $\Omega_i = \{\boldsymbol{\alpha}, p_i, y_i, h_{w_i}, \theta_{i,0}, G_i(\cdot)\}$.

The distribution $G_i(\cdot)$ has a mean equal to $E[\ln \theta_{i,1} | \Omega_i]$ and a variance equal to

[5] I will use "parent" and "mother" interchangeably, as this is a single-agent model.

$Var(\ln \theta_{i,1}|\Omega_i)$. Assume that: (i) $E[\xi_i|\Omega_i] = 0$; (ii) $E[\delta_k|\Omega_i] = \mu_{i,\delta_k}$ and $Var(\delta_k|\Omega_i) = \sigma^2_{i,\delta_k}$; (iii) $Cov(\delta_1, \delta_2|\Omega_i) = \sigma_{i,\delta_1,\delta_2}$ and $Cov(\delta_k, \delta_l|\Omega_i) = 0$ for all others $k \neq l$; and (iv) $Cov(\xi_i, \delta_k|\Omega_i) = 0$ for all k. In other words, parental beliefs about production shocks have mean zero and are uncorrelated with beliefs about the other parameters of the skill production function, and beliefs about δ_0, which defines the location of the baseline skill, are uncorrelated with beliefs about δ_1 and δ_2.[6]

Under these assumptions, we can write the *subjective* parental expectation and uncertainty about the skill of technology formation as

(4)

$$\mu_{i,\theta_1} \equiv E[\ln \theta_{i,1}|\Omega_i] = \mu_{i,\delta_0} + \mu_{i,\delta_1} \ln \theta_{i,0} + \mu_{i,\delta_2} \ln x_i,$$
$$\sigma^2_{i,\theta_1} \equiv Var(\ln \theta_{i,1}|\Omega_i) = (\sigma^2_{i,\delta_0} + \sigma^2_{i,\xi_i}) + \sigma^2_{i,\delta_1} \ln \theta^2_{i,0} + \sigma^2_{i,\delta_2} \ln x^2_{i,0} + \sigma_{i,\delta_1,\delta_2} \ln \theta_{i,0} \ln x_i.$$

These definitions derive directly form using the expectation and variance operators on (3). The parameter μ_{i,δ_2} is the parental subjective expectation of returns to investment, while σ^2_{i,δ_2} denotes the uncertainty about δ_2.

Parents maximize their expected utility function conditional on their information set:

$$\max_{x_i}\{E[u_i(c_i, h_{l_i}, \theta_{i,1}; \boldsymbol{\alpha})|\Omega_i]\}, \tag{5}$$

subject to the budget constraint (1), the time constraint (2), and the technology of skill formation (3). The optimal investment function depends on parental preferences, their income, the price of investment, their working hours, and on their subjective expectation and uncertainty of the returns to investment, μ_{i,δ_2} and σ^2_{i,δ_2}:

$$x^*_i = f(\boldsymbol{\alpha}, y_i, p_i, h_{w_i}, \mu_{i,\delta_2}, \sigma^2_{i,\delta_2}, \sigma_{i,\delta_1,\delta_2}).$$

Previous papers consider models that only depended on $\boldsymbol{\alpha}$, y_i, p_i and δ_2, which imply that parents have complete knowledge about the technology of skill formation (e.g., Del Boca, Flinn, and Wiswall (2014)). Without additional information about subjective beliefs, it is not possible to estimate a model that does not assume complete knowledge. Cunha, Elo, and Culhane (2013) and Attanasio, Cunha, and Jervis (2019) assume a Cobb-

[6]The assumption that $E[\xi_i|\Omega_i] = 0$ also implies that beliefs about shocks to the process are uncorrelated with covariates from the production function, $\ln \theta_0$ and x. As will be shown later, this assumption is satisfied due to the design of the survey.

Douglas utility function on consumption and child's skill which by construction removes the uncertainty of beliefs from the optimal investment function. This formulation of a parental investment model which incorporates subjective beliefs about the technology of skill production generalizes models in past research.

3. Methodological Framework

The objective is to elicit from parents their subjective expectation and uncertainty about the technology of skill formation function. There are two main challenges involved in this process. First, the variable that represents the human capital of the child, θ, is a latent variable with no meaningful metric. While there are many ways of dealing with this issue when using observational data, the added challenge in the context of elicitation of beliefs is that it is necessary to extract beliefs about these latent objects directly from individuals. Second, as can be seen from equation 4, the parameters are indexed by the individual which requires multiple observations for each individual of the relevant objects.

In the following subsection, I develop the survey instrument such that there is a clear mapping between a nationally representative metric of children's skills, θ, and an observable and easily interpretable variable to parents that is meaningful with respect to child development. Then, I develop an identification argument from this survey design and introduce an appropriate estimator.

3.1. Survey Instrument

Before going into detail on how I elicit the subjective belief distribution of parents, it is useful to explain how to measure the *objective* development of a child in a cardinal metric. The most common measure of early childhood development is through developmental milestone assessments by caregivers or clinics. Examples include the *Bayley Scales of Infant and Toddler Development*, the *Motor and Social Development Scale*, and the *Ages and Stages Questionnaires*. They all involve evaluating whether a child at a specific age can perform activities or have achieved milestones that are appropriate for their age. They provide standardized information on children's development across multiple domains, including motor, language, and cognitive development.

I follow Cunha, Elo, and Culhane (2022) to develop the belief elicitation module. I give a brief overview of their methodology. They use the Motor and Social Development (MSD) scale from the National Health and Nutrition Examination Study 1988 (NHANES).

The MSD asks mothers to answer 15 out of 48 questions regarding the motor, language, and numeracy development of their child, conditional on their age. For example, mothers with children ages 0 to 3 months answer different questions than mothers with children ages 22 to 47 months. Each question asks whether their child can perform a specific activity appropriate for their age. Moreover, the number of items a child is able to perform increases with their age, which is helpful in anchoring the latent child development.

They estimate an item response theory (IRT) model using the MSD instrument to fix the cardinal metric of development into developmental age, or "age-equivalent score". The IRT model has many uses. First, it reduces the dimensionality of the MSD module by estimating which items are most salient with respect to child development. Second, it creates a mapping between an observable variable, the MSD items, to the latent variable θ in a cardinal metric. Third, as shown later, it can be used to translate parent's answers to the belief instrument into the latent variable θ.

Let a_i denote the child i's age when their MSD answers are measured. Let κ_i denote child i's development relative to other children at the same age. Then, $\kappa_i = 0$ means that the child's development is typical for their age, $\kappa_i > 0$ means that they are advanced for their age, and $\kappa_i < 0$ means that they are delayed for their age. For each child i and MSD item j, denote the latent variable $d^*_{i,j}$ as:

$$d^*_{i,j} = b_{j,0} + b_{j,1}\left(\ln a_i + \frac{b_{j,2}}{b_{j,1}}\kappa_i\right) - \eta_{i,j}. \tag{6}$$

We observe $d_{i,j}$, which is a binary variable equal to 1 if $d^*_{i,j} \geq 0$ and 0 otherwise. The IRT model aims to estimate the latent factor κ_i. The parameter $b_{j,0}$ decreases with the difficulty of item j, while parameter $b_{j,1}$ increases as item j's difficulty decreases with age. Finally, parameter $b_{j,2}$ increases the more information item j contains about κ_i. Assuming that $\eta_{i,j}$ is i.i.d. normally distributed $N(0,1)$, then the probability that child i can perform activity j is given by

$$Prob(d_{i,j} = 1|a_i, \kappa_i) = \Phi(b_{j,0} + b_{j,1}\left(\ln a_i + \frac{b_{j,2}}{b_{j,1}}\kappa_i\right)).$$

This model can be estimated via maximum likelihood given suitable normalizations for κ_i.[7]

[7] In Cunha, Elo, and Culhane (2022), they assume that κ_i follows a mixture of two Normal distributions

This model provides an important piece to the development of the survey instrument. It allows me to have a nationally representative measure of child development in the easily interpretable metric of (the log of) *developmental age* in months. It also reveals which MSD items are the more meaningful in terms of explaining child development. Finally, given the answers from respondents, Equation 6 is used to translate parental beliefs into the estimated cardinal metric obtained from the IRT. The following subsections explain these features in more detail.

3.1.1. Subjective Belief Instrument

I now describe the process to elicit the subjective expectation and uncertainty of the technology of skill formation from parents. The main idea is to present parents with a hypothetical family and child and ask them their beliefs about the typical ages this child is able to perform certain activities under a specific level of initial human capital and parental investment. These activities are milestones that are used in the MSD-NHANES to assess developmental progress in children.

The general idea of the instrument is to tell the respondent to imagine a *hypothetical* scenario of initial human capital of the baby θ_0 and a level of investment x.[8] In designing belief elicitation exercises, there is an important trade-off between asking respondents about their own beliefs (self-beliefs) versus presenting them with a hypothetical family or scenario. Self-beliefs are often more economically relevant, as they reflect the actual decisions individuals face in their own lives, shaped by their unique circumstances and constraints. However, using hypothetical families offers several advantages. It simplifies the task, makes it easier to isolate and hold constant specific variables, and enables a clean comparison of beliefs across individuals, since everyone is responding to the exact same object. This comparability is particularly valuable when trying to identify patterns or regularities in belief formation or preferences.

Importantly, there is a conceptual link between self-beliefs and beliefs about hypothetical families. One can think of self-beliefs as a function of observable demographics, subjective beliefs about factors such as parenting ability and child receptivity, and expectations about a more general or representative case—that is, a hypothetical family. Empirical evidence supports a strong and positive relationship between self-beliefs and population-level or hypothetical beliefs in other domains, such as education and labor market expectations (see Bleemer and Zafar (2018); Wiswall and Zafar (2015)).

with mean zero. Additionally, to pin down the scale, they set $b_{2,j} = 1$ for one of the MSD items.

[8]The full instrument can be seen in Appendix A.

Moreover, in the context of elicitation of beliefs about returns to parental investment, there is evidence that beliefs about hypothetical families have strong predictive power about actual investment (see Cunha, Elo, and Culhane (2022); Attanasio, Cunha, and Jervis (2019); Attanasio, Boneva, and Rauh (2019); Biroli et al. (2022)). In the context of choice experiments, List, Sinha, and Taylor (2006) shows that respondent choices to contribute to a public good campaign are not statistically different between samples faced with a hypothetical setting and samples where they are asked to actually pay.

In my survey, I adopt a middle-ground approach. I ask respondents to consider a hypothetical family, but I deliberately leave its characteristics vague. This design choice reduces cognitive burden—respondents are not required to adjust their thinking to a detailed and potentially unfamiliar profile—while still preserving some degree of standardization. In doing so, the goal is to capture how individuals apply their beliefs in practice without anchoring them too rigidly to either their own personal situation or an overly stylized scenario.

The question that is asked to the respondent is the following:

What do you think are the youngest, most likely, and oldest ages a child will learn how to do [MSD ACTIVITY]?

Ideally I would ask parents about all milestones for 24 month-old children, but to avoid respondent fatigue I use the 4 most salient ones as estimated from the IRT model in 6 and used by Cunha, Elo, and Culhane (2022), namely[9]

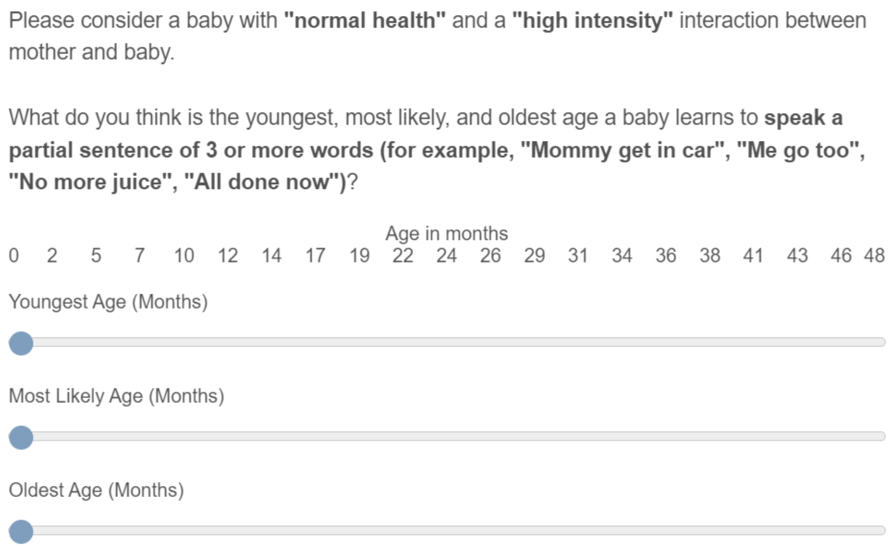
1. Speak a partial sentence of 3 words;
2. Count 3 objects correctly;
3. Say first and last name together;
4. Know age and sex.

The other important aspect is the definition of the hypothetical scenarios. I limit the scenarios to a 4-way combination of normal and poor initial health ($\bar{\theta}_0$ and $\underline{\theta}_0$ respectively) and high and low investment level ($\bar{x}$ and $\underline{x}$). I first describe to the respondent what does it mean for a baby to have normal or poor health in the context of this survey. A normal health baby is one whose gestation lasted 9 months and that weighed 8 pounds

[9]The parameters of the IRT model give estimates of the importance and relevance of each item in explaining the variability of the latent variable. By examining which items have the larger factor loadings estimates, the researcher can choose items with more explaining power. See Cunha, Elo, and Culhane (2013) for a detailed description of how these items were chosen.

and measured 20 inches in length. A poor health baby is one whose gestation lasted 7 months, weighed 5 pounds and measured 18 inches.[10]

FIGURE 1. Subjective Belief Instrument



Note: This figure shows the main survey instrument presented to respondents. It starts by outlining the hypothetical scenario to be considered, followed by the belief question. Highlighted in bold are the hypothetical scenario values and MSD activities. Respondents are restricted to give ascending order answers from youngest, most likely, and oldest ages.

Then, I describe in detail the difference between *active and passive interaction* between parent and baby. I follow an extensive literature that distinguishes the quality of interaction and define *active interaction* as one in which the parent's main focus is in the child.[11] I list several examples of active interaction to the parent, such as (i) soothing the baby when he/she is upset; (ii) playing peek-a-boo with the baby; (iii) singing songs with the baby; (iv) feeding, nursing, bathing, attending to health needs; among others. In contrast, I give examples of passive interaction with the baby, such as (i) grocery shopping with baby; (ii) browsing social media apps on smartphone with baby at your side; (iii) nap time for baby; (iv) household chores (cleaning, cooking, etc) while baby is

[10]These numbers are the same used in Cunha, Elo, and Culhane (2022), and were obtained from the Children of the National Longitudinal Survey of the Youth/1979 (CNLSY). They estimate a factor model using gestation, weight at birth, and height at birth as measures. The low scenario describes a premature birth: gestation lasts seven months (percentile 1 in the CNLSY/79), the birth weight is five pounds (percentile 4), and the length at birth is 18 inches (percentile 11). The normal scenario describes children born in a normal term. The gestation lasts nine months (percentile 85), weighs eight pounds (percentile 69), and the length at birth is 20 inches (percentile 85). Given factor score estimates and predicted scores, they compute the implied value of the latent variable θ_0 under a cardinal scale.

[11]See Folbre et al. (2005); Bono et al. (2016); Guryan, Hurst, and Kearney (2008); Schoonbroodt (2018).

at your side; among others.

I then define a high intensity interaction is one in which the parent spends 6 hours in active time, while a low intensity is one where the parent spends 2 hours.[12]

A scenario is a pair of either normal $\bar{\theta}_0$ or poor health $\underline{\theta}_0$, and high $\bar{x}$ or low $\underline{x}$ intensity interaction. Given a scenario (θ_0, x), the respondent is asked what they think is minimum, most likely, and maximum age in months that a baby will learn how to do activity j. They are presented with 3 sliders, one for each answer, that ranges from 0 to 48 months. The only restriction put in place is that the youngest age has to be less than or equal to most likely, which in turn has to be less than or equal to the oldest age. Figure 1 below depicts the instrument as shown to the respondents.

3.1.2. From age ranges to belief distributions

From the subjective belief instrument, I obtain in months the youngest age $\underline{a}_{i,j,k}$, most likely age $\dot{a}_{i,j,k}$, and oldest age $\bar{a}_{i,j,k}$ a respondent i believes a child under scenario k will learn how to do MSD item j. I assume that these replies are a random variable that follows a distribution $H_{a_{i,j,k}}(\cdot)$ with mean $\mu_{a_{i,j,k}}$ and variance $\sigma^2_{a_{i,j,k}}$. The mean $\mu_{a_{i,j,k}}$ represents the average age individual i believes a child will learn MSD item j under scenario k, while the variance $\sigma^2_{a_{i,j,k}}$ represents the degree of uncertainty from the individual about j under k.[13] When constructing the distribution $H_{a_{i,j,k}}(\cdot)$ from these answers, one must provide a mapping between these answers to specific points in the distribution. The youngest and oldest ages can represent either bounds on the support of the distribution or extreme percentiles of an infinite support distribution. For the most likely age, it can either represent the expected value, the median, or the mode.[14] [15]

[12] These numbers are the 25th and 90th percentile of the distribution of time spent with their child in the Panel Study of Income Dynamics time diaries. Differently from the initial health data, hours of investment is already a cardinal measure, so there is no need in transforming them.

[13] Note that this variance contains uncertainty information about when children achieve specific milestones and the natural variability of these milestones in the population.

[14] These assumptions are studied and tested in the context of developing countries by Delavande, Giné, and McKenzie (2011b,a). The authors show that minimum and maximum type of questions are consistent with representing either the 5th or 10th (90th or 95th) percentiles of subjective distributions. While they do not study specifically a question asking what is the most likely outcome, they do examine questions of the type "what do you expect". They show that interpreting this answer as a mean value has very poor fit with respect to implied means from subjective probability questions, and poor prediction power with respect to realized future outcomes in follow-up surveys, while median and modes exhibit much better performance.

[15] Cunha, Elo, and Culhane (2022) asks youngest and oldest ages in the same context and assumes that $\underline{a}_{i,j,k}$ and $\bar{a}_{i,j,k}$ are the 10th and 90th percentile of a Normal distribution, while providing robustness checks with alternative distributions and percentile choices. They show that there are no substantial qualitative differences in results. Attanasio and Kaufmann (2014), in a context of future earnings, assume

For the main results of this paper, I assume that the youngest and oldest ages represent the endpoints of a triangular distribution, while the most likely age represents the mode. [16] From here on, I omit the subscripts i, j, k for conciseness, and only include when necessary to avoid ambiguity.

Note that $H_{a_i}(\cdot)$ is not yet a distribution about beliefs of future child skills, θ_i. To accurately estimate what the subjective returns to investment are in terms of θ_i, a latent variable, we need to transform the age range responses distribution, $H_{a_i}(\cdot)$, to the human capital subjective belief distribution, $G_{i,\ln\theta_1}(\cdot)$, which has mean $E[\ln\theta_{i,1}|\Omega_i]$ and variance $Var(\ln\theta_{i,1}|\Omega_i)$. Before going into details, I give a brief example of the method.

Suppose that an individual has a distribution H_{a_i} for activity "walk 3 steps" such that its median is 20 months. That is, this person believes that 50% children learn to walk 3 steps at about 20 months. We can then use the IRT model, which gives us a mapping between the probability a child of a certain age can walk 3 steps to a nationally representative measure of the latent variable $\ln\theta_i$. That is, if most children learn how to walk 3 steps by 15 months, we know that this individual's belief corresponds to a below-median human capital level $\ln\theta_i$. We can use the information from the IRT to map beliefs about ages to beliefs about $\ln\theta_i$. For the variance, I do not use any population information since someone's belief is not related to the population heterogeneity in completing MSD items. In turn, I preserve the spread and shape implied by H_{a_i} to construct $G_{i,\ln\theta_1}(\cdot)$. I now describe the process in detail.

Each MSD item has different difficulties and in the population children learn to count 3 objects at a different ages than children learn to say their first and last name together. Therefore, we must take into account that parents giving the same age ranges about for different MSD items imply different estimates of $E[\ln\theta_1|\Omega_i]$.

Following Attanasio, Cunha, and Jervis (2019), first we use the IRT model to compute an estimate of human capital, $\ln\theta_{j,1}$, for which half of the children in the population can complete a specific activity j. To do so, we can use equation (6):

$$Prob(d_{i,j} = 1|\ln\theta_{i,1}) = \Phi(b_{j,0} + b_{j,1}\ln\theta_{i,1}),$$

to compute the implied $\ln\hat{\theta}_{j,1}$ when $Prob(d_{i,j} = 1|\ln\theta_{i,1}) = 0.5$. This can be done by inverting the equation. It is important to note that this estimate is in the *same metric* as respondents answers, which is the developmental age of the child.

Let $\text{Med}(a_{i,j,k})$ denote the median from $H_{a_i}(\cdot)$, that is, the median age that the parent

a triangular distribution and treat minimum and maximum as endpoints.

[16] More details can be found in the following section.

i believes children achieves activity j under scenario k. We can now define the parental subjective belief about developmental delay as $\delta_{i,j,k} = \mu_{a_{i,j,k}} - \theta_j$. Using this variable, we can compute a measure of belief about child development in the same metric as the IRT equation, that is, the developmental delay (or lead) in months relative to a target age: $\ln A + \delta_{i,j,k}$. Therefore, the measure of subjective expectation for parent i, MSD item j, under scenario k is given by

$$\ln \theta_{i,j,k,1} = \ln(A - \delta_{i,j,k}). \tag{7}$$

The only remaining piece is to choose the target age A.

In the case of the mean belief, I use the population distribution (through the IRT model) to change the location of the age completion distribution $H_{a_i}(\cdot)$ to reflect how different MSD items have different mean completion ages in the population. However, in the case of the variance of the belief distribution, I do not need to use the population distribution since the variance of the population distribution reflects the population heterogeneity in completing specific activities, which has no relation to the uncertainty from parents.

The variance $Var(\ln \theta_1|\Omega_i)$ is composed of two terms: (i) the subjective uncertainty about the nature of the returns to investment and to initial skill; (ii) and the belief about the heterogeneity of the child development process $Var(\varepsilon_i|\Omega_i)$. Similarly, the age range distribution G_{a_i} contains information about both components. Consider two parents $i = \{A, B\}$ that give the same answer for $\dot{a}_{i,j,k}$, the most likely age for MSD item j, but parent A believes that the minimum and maximum ages are 20 and 30, while B believes it is 18 and 32. The implied variance $\sigma^2_{a_{i,j,k}}$ for parent A will be smaller than for parent B, but it does not necessarily mean parent B is more uncertain than parent A. Parent B may believe that MSD item j is harder, and therefore there is more heterogeneity in the population about which age children are able to accomplish it. This heterogeneity is captured by the constant in equation (4), which includes $Var(\varepsilon_i|\Omega_i)$.

I use the Interquartile Range of the $H_a(\cdot)$ distribution to maintain the shape and dispersion from $H_a(\cdot)$ to $G_\theta(\cdot)$.[17] The Interquartile Range (IQR) is the difference between the 75th and 25th percentiles of the distribution. The IQR of a distribution H is defined as:

$$IQR_H = H^{-1}(0.75) - H^{-1}(0.25).$$

[17]The use of the Interquartile Range as a measure of dispersion of beliefs is common in the literature of household surveys. See Bruine de Bruin et al. (2023).

Then, to obtain the estimate $\sigma^2_{\theta_{i,1}}$ of $Var(\ln\theta_1|H_i)$, I compute the value of IQR for the distribution H_{a_i} and impose the IQR of distribution $G_{\theta_{i,1}}(\cdot)$ to be the same. Then, I can compute the variance of $G_{\ln\theta_{i,1}}(\cdot)$ implied by the estimate $\mu_{\theta_{i,1}}$ and $IQR_{G_{\theta_1}} \equiv IQR_{H_{a_i}}$.[18]

3.2. Identification and estimation of the subjective expectation and uncertainty from belief distribution

While the discussion in the previous sections referred to general distributions of regarding human capital and subjective beliefs, in practice I need to parametrize all distributions to achieve identification. Crucially, I need to define a specific distribution for: (i) $H_{a_{i,j,k}}(\cdot)$, the distribution of beliefs about the age a child will learn MSD item j under scenario k, and (ii) $G_i(\cdot)$, the distribution of beliefs about the future human capital $\ln\theta_1$.

I assume that $H_{a_{i,j,k}}(\cdot)$ is a triangular distribution with mean $\mu_{a_{i,j,k}}$ and variance $\sigma^2_{a_{i,j,k}}$. This assumption allows the mode and median of the distribution to range from anywhere within the support as opposed to the Normal distribution. A triangular distribution is defined by three points: $\underline{a}_{i,j,k}$, $\dot{a}_{i,j,k}$, and $\bar{a}_{i,j,k}$, which I map into the minimum, most likely, and maximum age a child will learn MSD item j under scenario k.

Meanwhile, I assume that $G_i(\cdot)$ is "approximately" a normal distribution with mean $E[\ln\theta_{i,1}|\Omega_i]$ and variance $Var[\ln\theta_{i,1}|\Omega_i]$. More specifically, I only impose that the IQR of $G_i(\cdot)$ behaves approximately like a Normal distribution: $IQR_G = 1.348\sigma$, which can be thought of the distribution being approximately symmetric and likely not too skewed.

I estimate the parameters of the subjective production function using different assumptions for the distribution of $H_{a_{i,j,k}}(\cdot)$. These results can be found in Appendix D.

In principle, one can directly estimate the parameters of interest, μ_{i,δ_2} and σ^2_{i,δ_2}, by computing the differences between the high and low investment scenarios ($\bar{x}$ and $\underline{x}$) conditional on a health level for $\mu_{\theta_{i,j,k,1}}$ and $\sigma^2_{\theta_{i,j,k,1}}$. However, this necessarily assumes that there is no measurement error in any of the measures that were collected.

I now show how to estimate μ_{i,δ_2} and σ^2_{i,δ_2} while addressing general measurement error. To ease on notation, define $T = J \times K$ as a new index that aggregates all possible combinations of activities j and scenarios k. Let $\varepsilon^{\mu}_{i,t} = (\varepsilon^{\mu}_{i,1}, \ldots, \varepsilon^{\mu}_{i,T})'$ and $\varepsilon^{\sigma}_{i,t} = (\varepsilon^{\sigma}_{i,1}, \ldots, \varepsilon^{\sigma}_{i,T})'$ denote the vector of measurement error associated with $\mu_{\theta_{i,t,1}}$ and $\sigma^2_{\theta_{i,t,1}}$ respectively. Define $z^{\mu}_{i,t} = (1.0, \ln\theta_{i,t,0}, \ln x_{i,t})$ and $z^{\sigma}_{i,t} = (1.0, (\ln\theta_{i,t,0})^2, (\ln x_{i,t})^2, \ln x_{i,t}\ln\theta_{i,t,0})$. Now,

[18] Under normality, the relationship between the IQR and variance is given by a simple formula: $IQR = 1.348\sigma$.

similarly aggregate $\mu_{\theta_{i,1}}$, $\sigma^2_{\theta_{i,1}}$, z_i^{μ} and z_i^{σ}. Finally, let $\beta_{i,\mu} = (\mu_{i,\delta_0}, \mu_{i,\delta_1}, \mu_{i,\delta_2})$ and $\beta_{i,\sigma} = (\sigma^2_{i,0}, \sigma^2_{i,\delta_1}, \sigma^2_{i,\delta_2}, \sigma_{i,\delta_1,\delta_2})$. We can then write a measurement error model:

$$\mu_{\theta_{i,1}} = E[\ln \theta_{i,1}|\Omega_i] + \varepsilon_i^{\mu} = z_i^{\mu}\mu_{i,\delta} + \varepsilon_i^{\mu}, \tag{8}$$

$$\sigma^2_{\theta_{i,1}} = Var(\ln \theta_{i,1}|\Omega_i) + \varepsilon_i^{\sigma} = z_i^{\sigma}\sigma_{i,\delta} + \varepsilon_i^{\sigma}. \tag{9}$$

Note that the system above is a system of linear regressions with an additional "panel" dimension and random coefficients. Given that both $\mu_{\theta_{i,1}}$, and $\sigma^2_{\theta_{i,1}}$ are measures constructed from the same set of responses, it is very likely that the measurement errors ε_i^{μ} and ε_i^{σ} are correlated to each other. Therefore, previous approaches to estimating these models, such as single-equation random coefficient models as in Swamy (1970) are not appropriate. Instead, I propose an estimator that accommodates a model of seemingly unrelated equations in a panel setting with random coefficients.[19]

To state the assumptions needed to identify the parameters of this model, it is useful to rewrite the model by first aggregating the two equations in a vector. Collect all equations such that $y_{i,t} = (\mu_{\theta_{i,t,1}}, \sigma^2_{\theta_{i,t,1}})'$, $Z_{i,t} = \begin{pmatrix} z_{i,t}^{\mu} & 0 \\ 0 & z_{i,t}^{\sigma} \end{pmatrix}$, $\beta_i = (\mu_{i,\delta}, \sigma_{i,\delta})$, and $\varepsilon_{i,t} = (\varepsilon_{i,t}^{\mu}, \varepsilon_{i,t}^{\sigma})$. Finally, define $y_i = (y_{i,1}, \ldots, y_{i,T})$, $Z_i = \begin{pmatrix} z_i^{\mu} & 0 \\ 0 & z_i^{\sigma} \end{pmatrix}$, $\varepsilon_i = (\varepsilon_{i,1}, \ldots, \varepsilon_{i,T})$, and $\beta_i = \beta + \eta_i$. Then, the system becomes:

$$y_i = Z_i(\beta + \eta_i) + \varepsilon_i;$$
$$y_i = Z_i\beta + (\varepsilon_i + Z_i\eta_i) = Z_i\beta + u_i,$$

Under this notation, I now introduce the following assumptions:

A. $E[\varepsilon_{i,t}|Z_{i,t}] = 0; E[\eta_i|Z_i] = 0;$

B. $E[\varepsilon_{i,t}\varepsilon'_{i,t}|Z_{i,t}] = \Omega_i; E[\eta_i\eta'_i|Z_i] = \Delta;$

C. $E[u_iu'_i|Z_i] = Z_i\Delta Z'_i + (I_T \otimes \Omega_i),$

where $\otimes$ is the Kronecker product between two matrices, and I_T is a $T \times T$ identity

[19] Balestra and Negassi (1992) develop an estimator in the context of simultaneous equations, where the dependent variable for all equations $g' \neq g$ appear in equation g on the right hand side.

matrix. Assumption A. is respected in this experimental setting since the covariates Z_i are exogenously manipulated and fixed for all individuals. Crucially, this does not impose that individuals are correct about their beliefs on average. Assumption B. allows the measurement error $\varepsilon_{i,t}$ to be correlated across equations and be person-specific. However, it rules out correlation of the measurement error within an individual. Assumption B. also allows for a non-diagonal covariance matrix of the random coefficients. Assumption C. follows directly from the two previous ones.[20]

I provide a detailed description of the estimator in Appendix C. However, I give an overview of the estimator in this section. The overall idea is to estimate the system using a Feasible Generalized Least Squares (FGLS) estimator. In the first step, compute individual-by-individual Seemingly Unrelated Regression estimates of the parameter vector. Using these estimates, compute feasible estimators of the covariance matrices of the measurement error and random coefficients. From this, one can construct a (FGLS) estimator for the population level β. To produce efficient estimates of the random coefficients β_i, the estimator uses the difference between the population level estimate and the individual level first stage estimates.[21] [22]

3.3. Stated choice instrument

I now describe the second instrument, intended to elicit the individual's opportunity cost of investment. The instrument consists of a series of stated choice experiments. I create a series of hypothetical scenarios of monthly household income, initial baby's health, and how many hours the individual spends at work. Then, I ask the respondent to answer a question: how much would they be willing to pay to spend one hour of leisure instead of taking care of the baby.

I describe a situation in which the parent would like to spend one hour away from the baby every weekday of the month. I highlight that this one hour would be for personal

[20]An alternative would be to allow a more flexible measurement error assumption that allows some form of correlation within individuals. However, there are important trade-offs to consider. The current estimator does not need to impose parametric restrictions on the distribution of the random coefficients and allows for arbitrary correlation both between the random coefficients and between measurement error of equations. A factor model approach that allows for more specific forms of measurement error correlation would substantially increase the computation costs of estimating this problem and would restrict the distribution of the random coefficients and of the measurement error.

[21]Note that the parameters of equation (9) are variances, and therefore need to be strictly positive. I impose a non-negativity constraint on parameters when estimating.

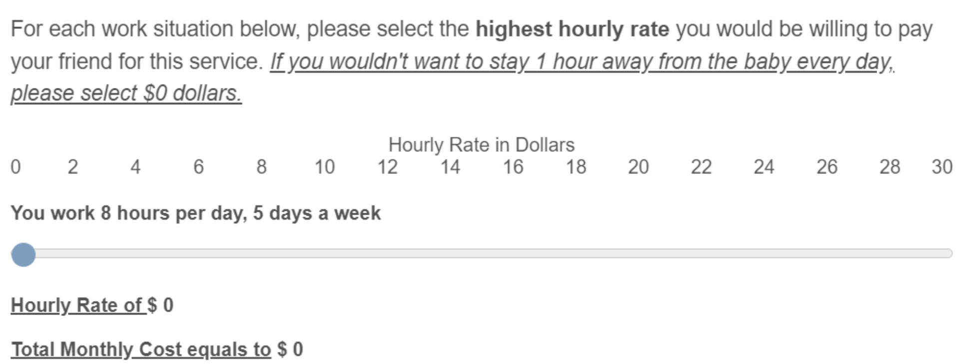
[22]The number of observations in each individual cross-section must be larger than the number of parameters to be estimated. There are 16 combinations of scenarios in two equations and 7 parameters to be estimated.

leisure. A friend offers to take care of the baby during this 1 hour, and while they will not be engaged in active interaction, the baby will be safe. I ask the individual to choose the highest hourly rate they would be willing to pay to their friend for the whole month. They choose out of a slider that ranges from $0 to $30. As they move the slider, they can see how much the hourly rate means in monthly expenses, assuming 20 weekdays in a month. If the person would rather not spend one hour away, I ask them to choose $0. The intention is to elicit their price of one hour of daily investment in a month. Also, it is important that the respondent does not see this extra hour of leisure as a perfect complement of child investment, such that there is an explicit tradeoff. Therefore, I emphasize that this is a friend and not a professional caretaker such as a nanny.

The exact wording of the questions are as follows:

> Think about the time you have available outside of work during weekdays. Imagine that for 1 month during weekdays (20 days), you want to spend 1 hour of leisure time away from your baby. A friend offers to take care of your baby during this 1 hour for the month, in exchange for a payment. Your friend will take good care of your baby, but they will not be engaged in active interaction with your baby.

FIGURE 2. Stated Choice Instrument - Willingness to Pay



For each question, I establish the following scenarios. The monthly household income can be $Y = \{\$2000, \$4000, \$6000\}$, the initial baby's health can be either $H = \{normal, poor\}$, and the working hours of the individual can be $HR = \{0, 4, 8\}$. These variables are chosen according to the model in Section 2.[23] The household income is

[23]The baby's initial health $\theta_{i,0}$ does not enter the decision function, so it can be used to test model implications.

chosen to reflect different percentiles of income in the population. A \$2,000 household income puts the household around the poverty line as established by the United States Federal Government, while a household income of \$6,000 puts the household around the median income.

3.4. Model Identification and Estimation

I describe the model identification and estimation strategy. I assume that the respondent's utility is linearly separable in the endogenous variables c_i, h_{l_i}, $\theta_{i,1}$. By following the steps described in the previous section, I can estimate for each individual the parameters of their subjective skill production function. I use them as inputs in the estimation procedure. The remaining parameters left are the preference parameters $\boldsymbol{\alpha}$, and the subjective price of one hour of investment, p_i.

Let k denote an element on the set of possible scenarios $K = Y \times H \times HR$. Additionally, denote by $p_{i,k}$ the maximum amount the individual i would pay their friend under scenario k. I asssume that $p_{i,k}$ is a noisy measure of the true subjective price of one hour of investment, p_i. I estimate p_i under a flexible factor model:

$$p_{i,k} = p_i + \boldsymbol{\gamma}_i x_k + \varepsilon_{i,k}, \tag{10}$$

where x_k denote the scenario variables, $\boldsymbol{\gamma}_i$ is the vector of coefficients associated with the scenarios, and $\varepsilon_{i,k}$ is a measurement error. I estimate the parameters of the model using the Swamy (1970) estimator.[24]

The optimal investment choice implied by the model is $x(\alpha)$, while the econometrician observes x^*. Due to measurement error, these values are different from each other. Let η_x denote the associated measurement error. Assume that:

$$x^* = x(\alpha)e^{\eta_x}, \tag{11}$$

where η_x is normally distributed with mean $\mu_{\eta_x} = -\frac{\sigma^2_{\eta_x}}{2}$ and variance $\sigma^2_{\eta_x}$.

Given these assumptions, the likelihood function is given by, where ϕ is the pdf of the standard normal distribution:

$$l(\alpha) = \sum_i^N \Big(\ln \phi(\ln x_i^* - \ln x_i(\alpha))\Big). \tag{12}$$

[24]Results of this estimation can be found in Appendix E.

To construct the model implied optimal investment choice $x(\alpha, p_i)$, one must compute the indirect utility function over a fine grid of possible values of x. I define the space of possible values of x as the interval $X = (0.0, 20.0]$[25] For a given value of $\alpha = \bar{\alpha}$, I construct a grid over this interval composed of 100 equidistant points, and compute the indirect utility function for each value. Then, I find the value $x^1(\bar{\alpha})$ which gives the maximum utility. I then construct a smaller interval around $x^1(\bar{\alpha})$ with 100 equidistant points, and proceed with the same algorithm and obtain $x^2(\bar{\alpha})$. I repeat this process until the difference between any two steps is smaller than 10^{-10}. This process is repeated for each new guess of α.

4. Data

I collect the data using Qualtrics, a company that holds online panels of individuals who are willing to take surveys for a small fee. The survey was conducted during April, 2023. My sample consists of 507 women between the ages of 18 and 40 who had at least one child, but no child older than 5 years old.[26] In this section, I present the answers for all survey segments and descriptive statistics. I also present evidence that each survey segment has consistent and economically meaningful participant answers.

4.1. Sample Characteristics and Actual Investment

The survey participants answer the questions from the subjective belief instrument and the stated choice instrument. Additionally, participants answer several demographic questions and report actual investment in their oldest child. Table 1 describes the sample characteristics. The average age of participants is 27.9, with about 1.4 children on average, while the average age of children is 2.2. The sample is also relatively well educated, with 43.4% of the sample having at least a 4 year college degree. Personal and household income are distributed evenly across the categories. The sample is also relatively diverse, with 22.3% of individuals being Hispanic, 27.4% being Non-Hispanic Black, and 39.1% being Non-Hispanic White. Around 79% of women are working on average 5.4 hours a day, while 32% are in school.

[25]Optimal investment is measured in terms of daily hours. The lower bound of the interval is set to 10^{-6}.

[26]The full sample consisted originally of 723 women. The number was reduced to 507 after removing individuals that did not pass attention checks, gave inconsistent answers, or chose to not answer some questions.

Table 1. Sample Composition

Variable	Mean	St. Dev.
Age of Respondent	27.876	5.678
Number of Children	1.377	0.611
Age of Oldest Child	2.203	1.295
Working	0.787	0.410
Daily Work Hours	5.400	3.625
In School	0.320	0.467
Ethnicity		
Hispanic	0.223	0.417
Non-Hispanic White	0.391	0.488
Non-Hispanic Black	0.274	0.447
Other	0.112	0.316
Marital Status		
Single	0.292	0.455
Married or Cohabitating	0.657	0.475
Separated	0.051	0.221
Education		
Dropout or GED	0.085	0.279
High School	0.162	0.369
Some College	0.320	0.467
College Degree	0.434	0.496
Household Income		
Less than $25,000	0.179	0.384
$25-$50,000	0.205	0.404
$50-$100,000	0.341	0.475
More than $100,000	0.274	0.447

Note: This table reports descriptive statistics for the sample of 507 respondents. All variables are proportions except for Age of Respondent, Number of Children, Age of Oldest Child, and Daily Work Hours. The Daily Work Hours variable is based only on respondents who reported being employed.

In Table 2, I present statistics of actual investment. The participants are asked how many hours they spend with their child on a typical weekday and weekend day on reading, talking, playing inside, and playing outside. Total hours spent is the sum of all 4 activities for each respondent. Table 2 shows that the average participant spends 2.9 hours on a typical weekday and 4.1 hours on a typical weekend day with their child. The average participant spends 0.4 hours reading, 1.1 hours talking, 0.9 hours playing inside, and 0.5 hours playing outside on a typical weekday. On a typical weekend day, the average participant spends 0.5 hours reading, 1.4 hours talking, 1.2 hours playing inside, and 0.9 hour playing outside. The table also shows that the standard deviation of the hours of investment is relatively large, suggesting that there is significant heterogeneity in the sample. These patterns show that parents spend more time with their child during weekends, and that talking is the most common activity.

TABLE 2. Actual Investment Statistics

Variable	Mean	Median	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
On A **Typical Weekday**, How Many Hours Do You Spend with your Child on							
Reading	0.365	0.250	0.461	0.000	0.033	0.500	5.883
Talking	1.083	0.500	1.469	0.000	0.083	1.500	8.333
Playing Inside	0.952	0.500	1.381	0.000	0.083	1.000	10.000
Playing Outside	0.517	0.333	0.660	0.000	0.033	0.833	6.000
Total	2.918	2.000	3.186	0.000	0.250	4.425	15.500
On A **Typical Weekend**, How Many Hours Do You Spend with your Child on							
Reading	0.544	0.333	0.725	0.000	0.033	0.750	6.000
Talking	1.443	0.667	1.909	0.000	0.083	2.000	10.000
Playing Inside	1.246	0.600	1.601	0.000	0.067	2.000	10.000
Playing Outside	0.874	0.500	1.177	0.000	0.050	1.000	10.667
Total	4.106	2.833	4.407	0.000	0.267	7.000	18.667

Note: This table reports descriptive statistics for actual investment measures based on a sample of 507 individuals. Respondents reported time spent (in minutes) on each activity during a typical weekday and weekend day. Total time reflects the sum of all reported activities. A small number of respondents were excluded due to implausible time reports. See Appendix B for more details.

To understand the determinants of investment, I estimate a linear regression of hours of total actual investment on individual's socio-economic variables. All continuous variables are standardized. The results are presented in Table 3. The table shows who those that currently work or go to school spend less time interacting with their child than those who do not. Additionally, the number of hours spent on a typical weekend is positively correlated with the number of hours worked during the weekday. This is consistent with the idea that parents face a time constraint on investment, but compensate on weekends by investing more.

Table 3 also shows that those with some college education or more spend more time interacting with their child than those with less education, while Non-Hispanic Black and Hispanics spend less time interacting with their child than Non-Hispanic Whites Finally, the table shows that those with a higher household income spend more time interacting with their child than those with a lower household income, although the impact is weak and not for all income brackets. Overall, the collected investment measures show patterns that are consistent with the literature: there is an richer, more educated parents invest more in their child, and there exists ethnicity differences in investment.

4.2. Subjective Belief Instrument Responses

I present survey participants‘ responses to the age range questions. Table 4 shows the mean and standard deviation of the youngest, most likely, and oldest ages for each activity asked for each scenario of investment and health. The activities are ordered by difficulty according to the IRT model. I highlight two data features. First, we see that the mean age responses tends to increase with the difficulty of the activity for all types of ages. This is consistent with participants paying attention to each activity and understanding that they have different difficulty levels for a child.

Second, we see that as the scenario moves from high investment and normal health to low investment and poor health, the mean age responses tend to increase. This is also seen in Figure 3, where I show histograms for the activity of speaking a partial sentence.[27] The histograms show that the distribution of age responses is shifted to the right as the scenario moves from high investment and normal health to low investment and poor health. This is consistent with the idea that participants are responding to the scenario and not just randomly answering the questions.

[27]The histograms for the other activities are similar and are presented in Appendix E.

TABLE 3. Correlations of Investment with Demographics

(a) Demographics – Part 1

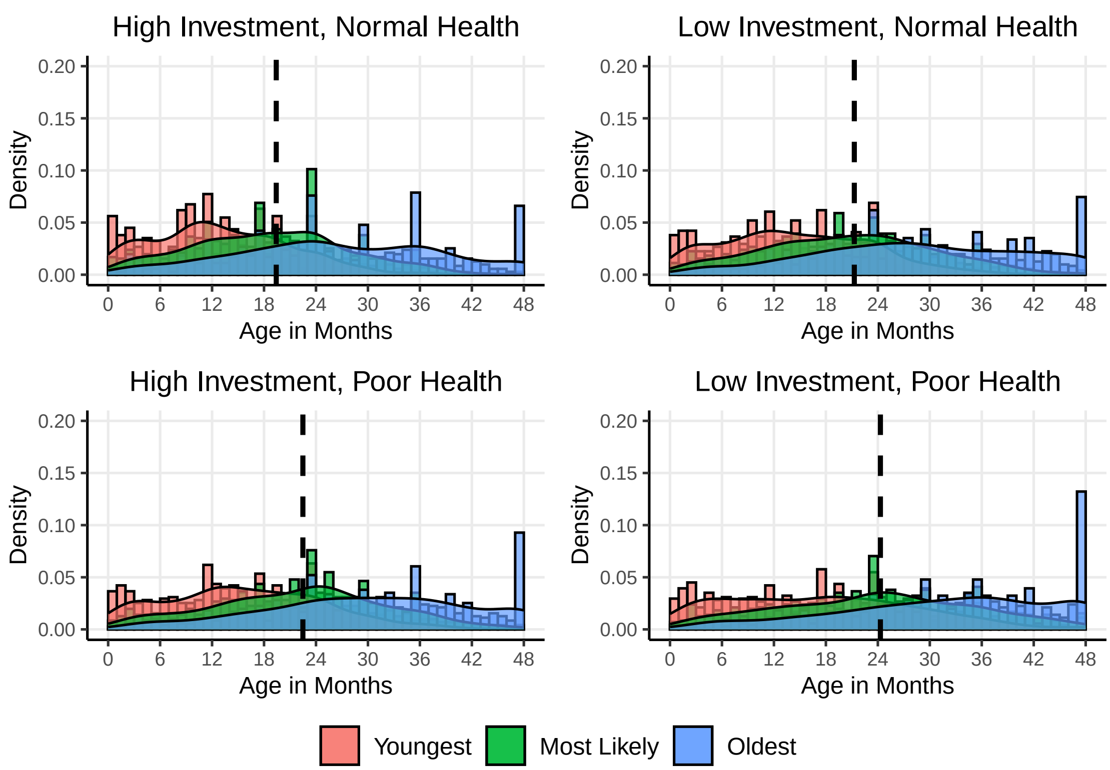
	Weekday Hours	Weekend Hours
Age ≤ 30	0.033	−0.120
	(0.103)	(0.109)
# Children	−0.004	−0.060
	(0.070)	(0.072)
Oldest Child ≤ 3	0.117	0.087
	(0.099)	(0.099)
Working	−0.599***	−0.486***
	(0.174)	(0.149)
Daily Work Hours	0.017	0.047***
	(0.019)	(0.017)
In School	−0.330***	−0.380***
	(0.099)	(0.100)
Ethnicity (Omitted: Non-Hisp White)		
Non-Hisp Black	−0.337***	−0.376***
	(0.115)	(0.120)
Hispanic	−0.154	−0.309***
	(0.118)	(0.111)
Other	−0.052	−0.242*
	(0.146)	(0.138)
Marital Status (Omitted: Single)		
Married	0.138	0.131
	(0.115)	(0.111)
Separated	0.010	−0.071
	(0.192)	(0.183)

(b) Demographics – Part 2

	Weekday Hours	Weekend Hours
Education (Omitted: Dropout)		
High School	−0.172	−0.065
	(0.181)	(0.158)
Some College	0.366**	0.424***
	(0.161)	(0.132)
College Degree	0.271*	0.320**
	(0.155)	(0.132)
Household Income (Omitted: $0-$25,000)		
$25-$50,000	0.291**	0.285**
	(0.141)	(0.123)
$50-$100,000	0.125	0.222*
	(0.125)	(0.119)
$100,000+	−0.065	0.033
	(0.147)	(0.139)
Constant	0.128	0.047
	(0.277)	(0.235)
Observations	507	507

Note: Robust standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01. This presents the estimates of regressing a typical weekday or weekend daily investment hours of a parent on their oldest child on socio-economic variables of the parent. The table is split into two for ease of view, but coefficients are all from the same linear regression. All continuous variables are standardized.

FIGURE 3. Distribution of Ages by Scenario for "Speak a Partial Sentence of 3 Words"



Note: This figure displays the histogram and density of the youngest, most likely, and oldest ages for the activity of speaking a partial sentence of 3 words. The histograms are colored by the type of age. The dashed line represents the mean of the most likely age.

Table 4. Mean and Standard Deviation of Responses to the Belief Instrument

	High Investment and Normal Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	13.12	7.96	18.67	9.24	26.44	11.99
Age and Sex	15.50	8.74	21.83	9.59	29.31	11.16
First and Last Name	18.39	10.61	24.99	10.79	32.04	11.87
Counts 3 Objects	16.83	10.11	23.19	10.59	30.84	12.15
	Low Investment and Normal Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	14.72	8.54	20.68	9.88	28.45	12.08
Age and Sex	17.41	9.56	23.91	10.17	30.86	11.47
First and Last Name	20.16	11.42	26.71	11.31	33.37	11.91
Counts 3 Objects	18.26	10.71	24.91	10.92	32.24	12.11
	High Investment and Poor Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	15.82	9.23	22.04	10.16	29.61	11.81
Age and Sex	18.04	10.12	24.46	10.37	31.67	11.56
First and Last Name	20.87	11.53	27.41	11.45	34.10	11.87
Counts 3 Objects	18.83	11.02	25.40	11.37	32.76	12.19
	Low Investment and Poor Health					
	Youngest Age		Most Likely Age		Oldest Age	
	Mean	SD	Mean	SD	Mean	SD
Partial Sentence	17.58	10.99	23.84	11.38	31.48	12.36
Age and Sex	20.48	11.83	26.98	11.73	34.07	12.25
First and Last Name	22.45	13.09	28.77	12.43	35.36	12.24
Counts 3 Objects	21.10	12.45	27.72	12.17	34.52	12.54

Note: This table presents the mean and standard deviation of the youngest, most likely, and oldest ages for each activity asked for each scenario of investment and health. The activities are ordered by difficulty according to the IRT model, with the easiest activity being “Partial Sentence” and the hardest being “Counts 3 Objects”.

4.3. Price of Investment Instrument

Table 5 displays descriptive statistics for the stated choice instrument. Respondents were asked to choose the maximum amount in dollars they would pay a friend to take care of their child. I present the mean and standard deviation for each scenario of health, household income, and daily working hours.

TABLE 5. Descriptive Statistics - Willingness-to-Pay

	Conditional on Good Health Household Income					
	$2,000		$4,000		$8,000	
	Mean	SD	Mean	SD	Mean	SD
Work 0 Hours	8.32	7.58	9.50	7.91	10.86	8.26
Work 4 Hours	11.29	6.90	12.41	6.51	13.47	6.90
Work 8 Hours	12.51	6.73	14.27	6.51	15.36	7.25
	Conditional on Poor Health Household Income					
	$2,000		$4,000		$8,000	
	Mean	SD	Mean	SD	Mean	SD
Work 0 Hours	9.12	8.44	9.74	8.30	11.22	8.95
Work 4 Hours	11.42	7.74	12.34	7.62	13.55	8.05
Work 8 Hours	12.58	8.13	13.56	7.95	15.05	8.69

Note: This table presents the mean and standard deviation of the maximum willingness-to-pay for the each scenario of health, household income, and daily working hours.

The table shows evidence of the consistency of the participants answers. The standard deviation in the sample across all scenarios remains constant, indicating that the variability in answers is similar regardless of the scenarios. There is no discernible difference across health scenarios. The means of each combination of work hour and income are similar whether the health of the baby is good or poor. This can be confirmed in Table 6, where the response is regressed on the scenario variables. The coefficients for baby health are all not statistically significant.

On both Tables, there is a strong gradient in work hours and household income. As they work more and earn more, their willingness-to-pay also increases. For example, conditional on good health, in the scenario of 0 working hours and $2,000 income, the mean response is of $8.32. Then, at 8 working hours, the mean response increases ot $12.51. Conversely, at $6,000 income, the mean increases to $10.86. This is again confirmed in Table 6. Increasing the working hours by 1 hour increases the willingness-to-pay by $0.52, while increasing the income by $1,000 increases the willingness-to-pay by $0.61.

These patterns are consistent with a model where parents value leisure. As their available hours of leisure decrease, they value it more. However, income is a significant factor, since as it increases, their budget constraint expands and they are able to allocate more resources to "buy" leisure. These correlations give support to the model presented in Section 2.

TABLE 6. Correlations of Willingness to Pay and Scenarios

	(1)	(2)	(3)	(4)
Hours of Work	0.52***			0.52***
	(0.03)			(0.03)
Household Income (in $ thousands)		0.61***		0.61***
		(0.04)		(0.04)
Baby Health			-0.06	-0.06
			(0.19)	(0.19)
Constant	10.18***	9.85***	12.30***	7.79***
	(0.27)	(0.28)	(0.27)	(0.35)
Observations	10,944	10,944	10,944	10,944

Note: Clustered standard errors at the individual level in parenthesis. *p<0.1; **p<0.05; ***p<0.01. The four columns show the correlation between willingness to pay and scenario variables. Hours of Work can be 0, 4, or 8. Household Income can be $2,000, $4,000, or $6,000. Baby Health can be good or poor.

5. Results

In this section, I first present the estimation results of the subjective production function parameters. Then, I discuss how they are related to each other, to demographic variables, and to actual investment measures. I then illustrate the use of parental subjective distributions by estimating an investment model with reference-dependent preferences, and discuss its interpretation.

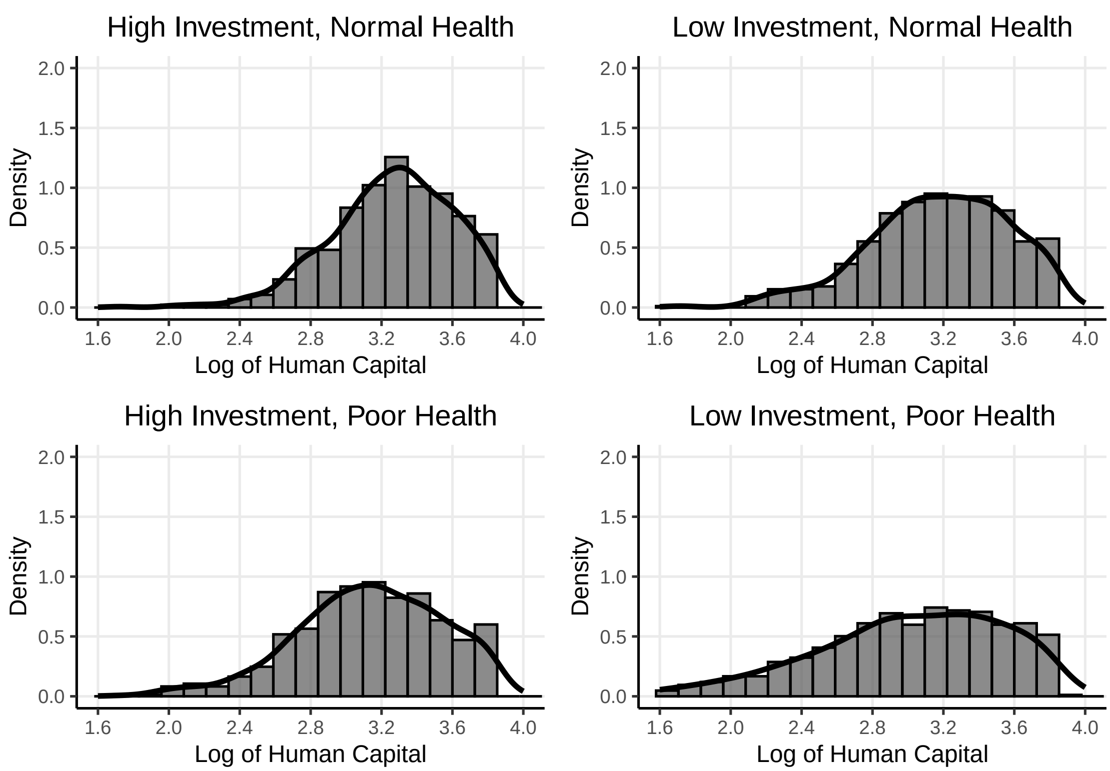
5.1. Parental Beliefs

I now present the estimates of the subjective expectation and uncertainty about the future human capital of the child. These estimates assume that the age range distribution

$H(\cdot)$ follows a triangular distribution.[28]

In Figure 4, I plot the histogram and distribution of $E[\ln\theta_{i,1}|\Omega_i]$ for each of the four possible hypothetical scenarios. For each scenario, there are four estimates of $E[\ln\theta_{i,1}|\Omega_i]$, one for each activity j. Therefore, I average $E[\ln\theta_{i,1}|\Omega_i]$ across all four activities. I find that the distribution of $E[\ln\theta_{i,1}|\Omega_i]$ is more dispersed and has a lower mean for the low investment and poor health scenario, while it is more concentrated around a higher mean for the high investment and normal health scenario. This indicates that parental beliefs are positively correlated with investment and health, as predicted by the model.

FIGURE 4. Distribution of Error-ridden Expectation of Log of Human Capital by Scenario



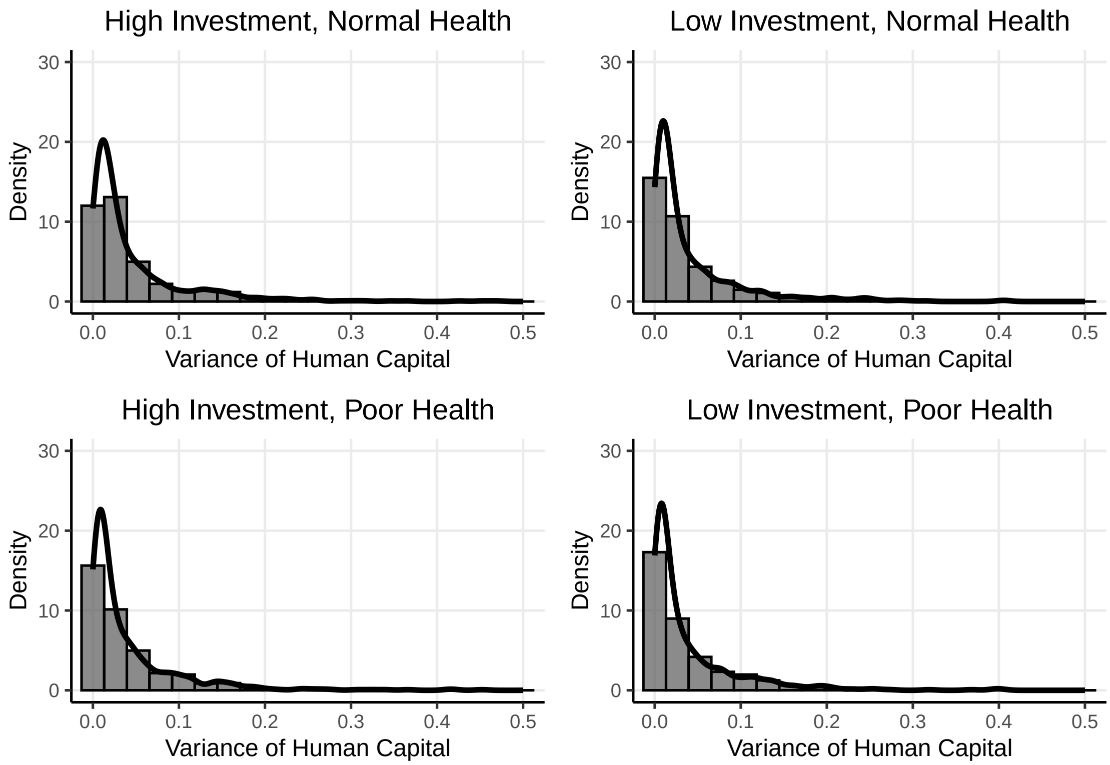
Note: This figure plots the histogram of the error ridden measures of $E[\ln\theta_{i,1}|H_i]$ conditional on the hypothetical scenario that was presented to the respondent.

In Figure 5, I plot the distribution of the error-ridden variance of the log of human capital. I find that the distribution is more concentrated for the low investment and poor health scenario, while it is more dispersed around a lower mean for the high investment and normal health scenario. Therefore, parents are more uncertain about the future

[28] Results for different combinations of distributional assumptions can be found in Appendix D.

human capital of their child under a scenario of higher returns.

FIGURE 5. Distribution of Error-ridden Variance of Log of Human Capital by Scenario



Note: This figure plots the histogram of the error ridden measures of $Var(\ln \theta_{i,1}|H_i)$ conditional on the hypothetical scenario that was presented to the respondent.

TABLE 7. Summary Statistics for $E[\ln \theta_{i,1}|H_i]$ and $Var(\ln \theta_{i,1}|H_i)$ by Scenario

	$E[\ln \theta_{i,1}|H_i]$		$Var(\ln \theta_{i,1}|H_i)$	
Scenario	Mean	St. Dev.	Mean	St. Dev.
High Investment, Normal Health	3.260	0.341	0.049	0.075
Low Investment, Normal Health	3.175	0.392	0.043	0.065
High Investment, Poor Health	3.131	0.414	0.044	0.069
Low Investment, Poor Health	2.990	0.562	0.041	0.062

Note: This table shows summary statistics for $E[\ln \theta_{i,1}|H_i]$ and $Var(\ln \theta_{i,1}|H_i)$ conditional on the scenario that was presented to the respondent.

Table 7 shows the mean and standard deviation of $E[\ln \theta_{i,1}|\Omega_i]$ and $Var(\ln \theta_{i,1}|\Omega_i)$

and summarizes the above discussion. The mean values of $E[\ln \theta_{i,1}|\Omega_i]$ decrease as the scenarios move from the best to the worst, while its standard deviation increases. For the "best" scenario, where there is high investment and normal health, parents believe that the child's skill will be 3.260, which translates to a developmental age of 26 months. Note that the skill is anchored at the age of 24 months, so parents are slightly optimistic under this scenario. For the "worst" scenario of low investment and poor health, parents believe the child's skill will be 2.990, or 19.8 months, a developmental delay of about 4 months.

In contrast, the mean and standard deviation of $Var(\ln \theta_{i,1}|\Omega_i)$ decrease as the scenarios move from the best to the worst. This is consistent with parents having more pessimistic and less uncertain beliefs about the future human capital of the child when investment is lower and when the child is in poor health.

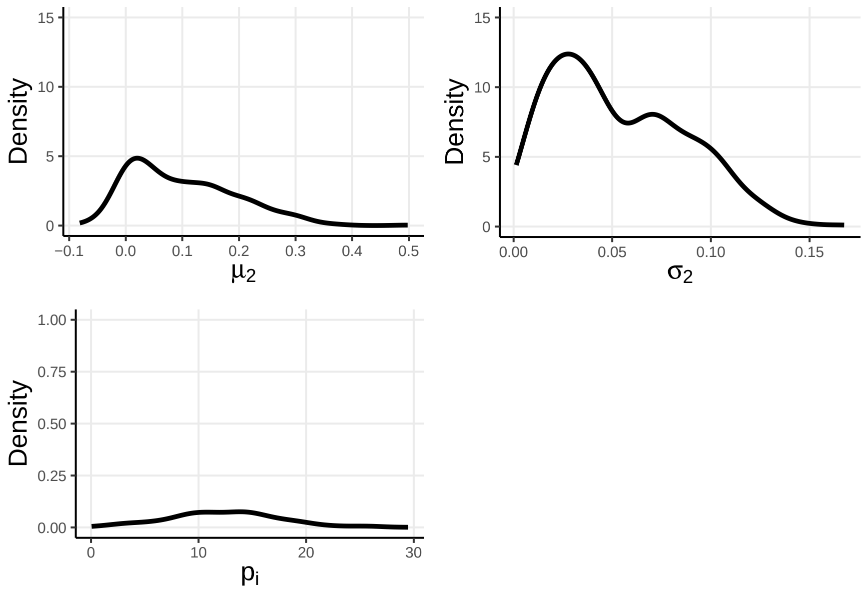
5.2. Estimates of the Subjective Production Function and Subjective Cost

Table 8 presents the estimates of the subjective production function parameters, μ_{δ_k} and $\sigma^2_{\delta_k}$, from (8) and (9) using the SRC estimator. Additionally, I estimate individual-level coefficients, μ_{i,δ_k} and σ^2_{i,δ_k}, and their standard errors.

Panel A of Table 8 displays the aggregate estimates. All parameters are statistically significant at the 1% level. I focus the discussion on the estimates of the subjective returns to investment parameters, δ_2, as they are the most relevant for the analysis. I find that the subjective mean of the returns to investment parameter, i.e. μ_{i,δ_2}, is 0.103. On average mothers believe that a 10% increase in investments would lead to a 1.03% increase in the child human capital by 24 months. As a comparison, Cunha, Elo, and Culhane (2022) report a subjective elasticity of investment of around 0.17, while Cunha, Elo, and Culhane (2013) report an objective elasticity of investment of around 0.26 using similar methods. Therefore, mothers in this sample are more pessimistic about the returns to investment than the previous literature. The estimated subjective variance, $\sigma^2_{\delta_2}$, is 0.003. Together with the mean estimate, these parameters give a coefficient of variation ($CV = \frac{\sigma}{\mu}$) of 0.53 thereby suggesting a low degree of uncertainty in beliefs of the average parent in the sample. At the individual level, only about 29% of the sample have a coefficient of variation higher than 1.0.

In Panel B, I conduct a significance test for each individual-level parameter. I report the percentage of the estimates whose p-values are lower than 10% confidence region. Focusing on the subjective returns to investment parameters, I find that 43.20% of the estimates of μ_{i,δ_2} are statistically significant at the 10% level, while 27.22% of the

FIGURE 6. Distribution of Individual Level Coefficients



Note: This figure displays the distribution of the efficient individual level estimates of the subjective expected returns to investment μ_{i,δ_2} and subjective uncertainty of returns to investment σ_{i,δ_2}.

estimates of σ^2_{i,δ_2} are statistically significant at the 10% level. The percentage value for the mean estimates are in line with Cunha, Elo, and Culhane (2022).[29]

Figures 6 and Table 9 display the distribution and summary statistics of the estimated individual level parameters of the subjective beliefs of the production function and the subjective costs.[30] The mean values of the production function parameters are by construction equal to the estimates in Table 8. The standard deviation and selected percentiles of the distribution of the individual-level parameters are also reported and

[29]The percentage of significant coefficients for the variance estimates is low. This is likely due to the fact that the variance estimates are more affected by measurement errors that are not fully corrected by the methodology I propose. On the other hand, it could also be due to model misspecification. For example, the full variance specification, assuming uncorrelation of production shock beliefs to all other parameters, would be:

$$Var(\ln \theta_{i,1}|H_i) = (\sigma^2_{i,\delta_0} + \sigma^2_{i,\varepsilon}) + \sigma^2_{i,\delta_1} \ln \theta^2_{i,0} + \sigma^2_{i,\delta_2} \ln x_i^2 + \sigma_{\delta_0,\delta_1} \ln \theta_{i,0} + \sigma_{\delta_0,\delta_2} \ln x_i + \sigma_{\delta_1,\delta_2} \ln \theta_{i,0} \ln x_i.$$

The estimation of this model is not feasible due to the nature of the estimation strategy. While theoretically feasible, the scenarios ($Z = \{\theta_0, x\}$) are simplified to be variables with only two distinct values and thus the within-individual variation of right-hand side variables is very small. Therefore, there would be severe multicollinearity in estimating the full model. Nevertheless, as I show in the next section, these estimates contain information that is relevant to demographic variables and real investment.

[30]I present the estimates of the factor model that produces the price coefficients in Appendix E.

TABLE 8. Estimates of Mean Subjective Production Function Parameters

Panel A: Aggregate Estimates			
$E[\ln \theta_{i,1} \mid H_i]$		$Var(\ln \theta_{i,1} \mid H_i)$	
μ_{δ_0}	2.897***	$\sigma^2_{\delta_0}$	0.031***
	(0.0285)		(0.0023)
μ_{δ_1}	0.064***	$\sigma^2_{\delta_1}$	0.001***
	(0.0045)		(0.0002)
μ_{δ_2}	0.103***	$\sigma^2_{\delta_2}$	0.003***
	(0.0084)		(0.0006)
		$\sigma_{1,2}$	−0.002***
			(0.0006)

Panel B: Individual Estimates			
Parameter	% Significant	Parameter	% Significant
μ_{i,δ_0}	100.00%	σ^2_{i,δ_0}	81.46%
μ_{i,δ_1}	45.17%	σ^2_{i,δ_1}	28.80%
μ_{i,δ_2}	43.20%	σ^2_{i,δ_2}	27.22%
		$\sigma_{i,1,2}$	16.17%

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows aggregate estimates of equations (8) and (9) and their individual-level predicted estimates. The percentage of significant estimates is calculated using a 10% significance level, and the null hypothesis is that the coefficient is equal to zero.

give an idea of the heterogeneity in beliefs. Focusing on the parameters related to the returns to investment, i.e. δ_2, I find that in general the mean beliefs exhibit variation across individuals, with the standard deviation of μ_{i,δ_2} being 0.094 and a coefficient of variation close to 1.0. The standard deviation of the variance estimates, σ^2_{i,δ_2}, is 0.004, which produces a similar coefficient of variation.

The mean price estimate shows that on average mothers price one hour of investment compared to leisure at around \$12.41, with the overall distribution of prices not being very dispersed, as the 25th and 75 percentiles are close to the mean. It implies a monthly cost of 1 hour care of children everyday on weekdays at around \$273.

TABLE 9. Individual Level Coefficient Distributions

Variable	Mean	St. Dev.	25th Percentile	Median	75th Percentile
p_i	12.410	5.178	9.018	12.358	15.607
μ_{i,δ_0}	2.916	0.570	2.527	2.924	3.389
μ_{i,δ_1}	0.063	0.055	0.019	0.053	0.098
μ_{i,δ_2}	0.100	0.094	0.022	0.086	0.163
$\sigma^2_{i,0}$	0.033	0.039	0.005	0.016	0.049
σ^2_{i,δ_1}	0.001	0.001	0.0001	0.001	0.002
σ^2_{i,δ_2}	0.004	0.004	0.0004	0.002	0.006
$\sigma_{i,\delta_1,\delta_2}$	−0.002	0.004	−0.004	−0.001	0.0003

Note: This table shows certain moments from the distribution of individual level coefficients. Price p_i was estimated using a linear regression random coefficients model, while the production function subjective parameters were estimated using the seemingly unrelated random coefficients model.

5.3. Relationship between Subjective Variables, Demographics, and Actual Investment

I now investigate how observable characteristics of mothers relate to their subjective beliefs. I focus on the subjective returns to investment parameters, μ_{i,δ_2} and σ^2_{i,δ_2}, and the price of care, p_i. I standardize these variables to have a mean equal to 0 and a standard deviation equal to 1 to keep the relationships comparable. Then I run a linear regression on the demographic variables described in the Data section.[31] I report the results in Table 10.

First, I find that younger mothers tend to have lower mean beliefs and to be more uncertain about the return to investment in children. Second, Non-hispanic Black mothers are more pessimistic and more uncertain about the return to investment than White mothers. Third, I find that married mothers are less uncertain about the return to investment, and price investments much lower than non-married ones. Fourth, I find a strong education gradient in mean beliefs. Finally, while income is not significantly correlated with beliefs, there is a strong income gradient in the cost of care.

While no causal relations can be extracted from these results, they suggest certain patterns that are consistent with the literature. For example, the racial difference in mean beliefs is consistent with the findings of Cunha (2014). The positive education gradient in beliefs is conceptually similar to the use of parental education as a proxy for parental knowledge of the benefits of early investment. Lower and more uncertain beliefs for younger mothers suggests that mothers learn throughout the process of raising children, although an extension incorporating learning in the production function is beyond the scope of this paper. Similarly, it is not surprising that the price of care is more costly for higher income mothers, as they are more likely to be employed and have higher opportunity costs of time. Finally, the negative correlation between marriage and uncertainty and price shows that family structure can be an important determinant in belief formation. This link has been noted in the literature on children's outcomes.

Next, I explore how the estimated beliefs relate to each other. I regress the subjective mean on subjective uncertainty and cost of care, and control for the observable characteristics. I also estimate the same model but using the subjective uncertainty as the dependent variable to obtain the correlation between cost of care as well. Table 11 reports the results. I find that mothers that have lower beliefs about the returns to investment tend to be more uncertain as well. This result holds as we add demographic controls and the price of care. Further, mothers that have higher costs of care tend to have lower mean beliefs.

Finally, I explore how the estimated beliefs relate to actual investment. Two different measures of time investment are collected in the data, the number of hours spent with the child on a typical weekday and weekend. Therefore, I estimate two regressions

[31] I exclude labor supply variables in this regression as they are "transitory" variables that do not reflect fixed characteristics of individuals. However, results do not change qualitatively when included.

TABLE 10. Correlation of Subjective Variables and Socio-Economic Variables

	μ_{i,δ_2}	σ^2_{i,δ_2}	p_i
Age ≤ 30	−0.234**	0.200**	0.046
	(0.107)	(0.090)	(0.101)
# Children	−0.096	0.219***	0.018
	(0.067)	(0.083)	(0.077)
Oldest Child ≤ 3	−0.052	0.163	−0.144
	(0.114)	(0.102)	(0.123)
Ethnicity (Omitted Group: Non-Hisp White)			
Non-Hisp Black	−0.267**	0.362***	0.062
	(0.123)	(0.131)	(0.123)
Hispanic	−0.288**	0.139	0.110
	(0.118)	(0.108)	(0.128)
Other	−0.060	0.058	−0.075
	(0.145)	(0.125)	(0.135)
Marital State (Omitted Group: Single)			
Married	0.140	−0.255**	−0.391***
	(0.109)	(0.113)	(0.117)
Separated	−0.094	−0.053	−0.060
	(0.203)	(0.235)	(0.213)
Education Level (Omitted Group: Dropout/GED)			
High School	0.153	−0.094	0.003
	(0.161)	(0.182)	(0.212)
Some College	0.461***	−0.148	−0.096
	(0.157)	(0.179)	(0.195)
College Degree	0.267*	−0.052	0.022
	(0.161)	(0.183)	(0.203)
Household Income (Omitted Group: $0-$25,000)			
$25-$50,000	0.193	−0.060	0.260*
	(0.140)	(0.154)	(0.158)
$50-$100,000	0.149	−0.046	0.338**
	(0.140)	(0.144)	(0.156)
$100,000+	−0.008	−0.156	0.360**
	(0.168)	(0.162)	(0.176)
Constant	0.003	−0.371	0.042
	(0.240)	(0.264)	(0.268)
Observations	507	507	507

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. All continuous variables are standardized. This table presents estimates of a linear regression model where the dependent variable is the mean and variance of the belief distribution about returns to investment. Independent variables consist of socio-economic indicators.

TABLE 11. Correlation between beliefs and cost of care

		μ_{δ_2}		
	(1)	(2)	(3)	(4)
σ^2_{i,δ_2}	-0.431^{***}		-0.411^{***}	-0.381^{***}
	(0.034)		(0.034)	(0.036)
p_i		-0.201^{***}	-0.142^{***}	-0.139^{***}
		(0.042)	(0.040)	(0.041)
	$\sigma^2_{\delta_2}$			
	(1)	(2)		
μ_{i,δ_2}	-0.431^{***}		-0.419^{***}	-0.379^{***}
	(0.040)		(0.041)	(0.042)
p_i		0.143^{***}	0.058	0.051
		(0.039)	(0.038)	(0.038)
Observations	507	507	507	507
Demographics	No	No	No	Yes

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. All continuous variables are standardized. This table presents estimates of a linear regression model where the dependent variable is the mean and variance of the belief distribution about returns to investment. Independent variables consist of beliefs and socio-economic indicators.

using these two measures as the dependent variables, and controlling for observable variables. Table 12 reports the correlation between the subjective returns to investment parameters and the actual investment measures. The first column uses only the mean belief as the independent variable, as in the previous literature, and I progressively add the subjective variance, the price of care, and demographic controls.

TABLE 12. Correlation of Real Investment with Beliefs

	(1)	(2)	(3)	(4)
		Weekday Hours		
μ_{i,δ_2}	0.353***	0.288***	0.260***	0.197***
	(0.042)	(0.049)	(0.049)	(0.052)
σ^2_{i,δ_2}		−0.153***	−0.142***	−0.132***
		(0.041)	(0.040)	(0.045)
p_i			−0.160***	−0.133***
			(0.040)	(0.039)
		Weekend Hours		
μ_{i,δ_2}	0.390***	0.291***	0.265***	0.200***
	(0.041)	(0.050)	(0.049)	(0.052)
σ^2_{i,δ_2}		−0.229***	−0.219***	−0.178***
		(0.041)	(0.040)	(0.042)
p_i			−0.153***	−0.146***
			(0.040)	(0.039)
Observations	507	507	507	507
Demographics	No	No	No	Yes

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. All continuous variables are standardized. This table presents estimates of a linear regression model where the dependent variable consists of actual time investment measures as reported by individuals. Each column adds additional controls such as belief distribution parameters and socio-economic indicators.

I find that the subjective mean of the returns to investment parameter, i.e. μ_{δ_2}, is positively correlated with the actual investment measures. This finding is consistent with the previous literature (see, Cunha, Elo, and Culhane (2013, 2022); Boneva and Rauh (2018)). A one standard deviation increase in mean beliefs leads to a 23% standard deviation increase in daily hours of investment, whether on weekdays or weekends. However, higher uncertainty predicts lower investments, with a one standard deviation increase associated with a 14% standard deviation decrease in investments. We see a similar pattern in the price of care, but the reduction of investment is larger for weekends than for weekdays.

Overall, these patterns provide reassurance that the elicitation methods provided meaningful data since results for mean beliefs follow the previous literature. Furthermore, the results for the subjective uncertainty suggests that increased uncertainty may play an important role in influencing investment, and future interventions that target belief improvements may find useful to also measure uncertainty.

5.4. An Application to a Model With Reference Dependent Preferences

Previous research either ignore subjective beliefs or assume specific functional forms for the utility function that rules out higher order beliefs. The stated choice data together with the subjective belief estimates for individuals allows me to estimate a flexible model of parental investment in children. In this section, I estimate a model of parental investment in children with reference dependent preferences that incorporates subjective uncertainty on the decision making of individuals. Parental preferences depend on household consumption, leisure, child development at the end of the period, and the relative development of the child compared to a reference level of development, θ_{ref}. I assume that the mother's preferences are given by:

$$\begin{aligned} u_i(c_i, h_{l_i}, \theta_{i,1}) = &\alpha_1 \ln c_i + \alpha_2 \ln h_{l_i} + \alpha_3 \ln \theta_{i,1} + \\ &\alpha_4(\ln \theta_{ref} - \ln \theta_{i,1})\mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\}. \end{aligned}$$

The parameter α_1 captures the preference for consumption and α_2 captures the preference for leisure. The coefficient α_3 captures how parents value the end-of-period human capital of their child, while α_4 denotes how parents value their child's skill being below the reference point. It is reasonable to assume that $\alpha_3 > 0$, since parents would like their child's human capital to increase. However, it is important to look at both α_3 and α_4. Since the reference point introduces a sharp discontinuity in θ_1, the sign and magnitude of α_4 relative to α_3 will determine how much parents value human capital beyond the reference point. If $\alpha_3 > \alpha_4 > 0$, parents will invest more after crossing the reference point. On the other hand, if $\alpha_3 > 0$, but $\alpha_4 < 0$, parents will have a strong incentive to invest only up to the reference point, and have a weak preference for human capital beyond the reference.

The reference point θ_{ref} is the level of development that the parents consider to be "desirable" or "satisfactory". In a similar context of parental child investment, Wang et al. (2022) and Kinsler and Pavan (2021) show that parents use their local peers as a reference point[32] In my context, this definition of a "local" peer is not feasible. Since child development is anchored in developmental age, a natural reference point is a threshold for developmental delay.

The expected utility function which parents maximize is:

$$E[u_i(c_i, h_{l_i}, \theta_{i,1})|\Omega_i] = \alpha_1 \ln c_i + \alpha_2 \ln h_{l_i} + \alpha_3 E[\ln \theta_{i,1}|\Omega_i] + \alpha_4 \int_{\theta_{i,1}} (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\} dG_i,$$

where $G_i(\cdot)$ is the distribution of subjective beliefs. Given the assumption that $G_i(\cdot)$ is a Normal distribution, it follows that the integral can be simplified to:[33]

$$\int_{\theta_{i,1}} (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\} dG_i = (\ln \theta_{ref} - \mu_{i,\theta_1}) \Phi\left(\frac{\ln \theta_{ref} - \mu_{i,\theta_1}}{\sigma_{i,\theta_1}}\right) + \sigma_{i,\theta_1} \phi\left(\frac{\ln \theta_{ref} - \mu_{i,\theta_1}}{\sigma_{i,\theta_1}}\right),$$

where Φ and ϕ denote the cdf and pdf of a standard Normal distribution, respectively, and μ_{i,θ_1} and σ_{i,θ_1} are the mean and standard deviation of the subjective beliefs of the production function. Then, the estimation of the model parameters $\boldsymbol{\alpha}$ follows the procedure described in section 3.4.

Table 13 displays the estimation of the preference parameters of the investment model.[34] Note that I use the subjective production function parameters and the subjective cost of care as inputs in the estimation procedure. Therefore, the estimates of the preference parameters take into account that different individuals face different costs of care and have different beliefs about the returns to investment.

I estimate the model for different reference points θ_{ref}. The elicitation of the sub-

[32] In Wang et al. (2022), the reference is the average development of children in the village, while in Kinsler and Pavan (2021) it is the peers in the same school and classroom. In both papers, parental beliefs are related to what the reference point is, not about the skill production function.

[33] Note that the integral involves the expected value of a Truncated Normal Distribution. Specifically,

$$\int_{\theta_{i,1}} (\ln \theta_{ref} - \ln \theta_{i,1}) \mathbb{1}\{(\ln \theta_{i,1} \leq \ln \theta_{ref})\} dG_i = E[\ln \theta_{ref} - \ln \theta_{i,1}|\Omega_i, \ln \theta_{i,1} \leq \ln \theta_{ref}] Pr(\ln \theta_{i,1} \leq \ln \theta_{ref})$$

.

[34] The sample used to estimate this model excludes 54 individuals that violate some model constraints. More details can be found in Appendix B.

TABLE 13. Model Estimates

Parameter	Estimates			
	θ_{ref} = 18	θ_{ref} = 20	θ_{ref} = 22	θ_{ref} = 24
α_2	1.132***	1.090***	0.555***	0.674***
	(0.354)	(0.375)	(0.176)	(0.275)
α_3	3.137***	2.071***	1.183***	1.282***
	(0.745)	(0.525)	(0.239)	(0.311)
α_4	-3.077***	-2.031***	-1.159***	-1.262***
	(0.684)	(0.487)	(0.219)	(0.291)

Note: Bootstrapped standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows the estimates of the preference parameters of the investment model for different reference points. α_2 denotes the preference parameter for leisure, α_3 for the child human capital, and α_4 for the positive distance between the child's human capital and the reference point when it would fall below the reference.

jective beliefs targets the developmental age of 24 months. In other words, the activities that are used to elicit beliefs from parents were typical for children around 2 years old, and the anchoring of the beliefs to a cardinal metric used the 24 months milestone. Then, the reference point θ_{ref} should be a developmental age around 24 months. I estimate the model using $\theta_{ref} = \{18, 20, 22, 24\}$ months. A $\theta_{ref} = 18$ indicates that the reference point is a developmental age of 18 months, which represents a developmental delay of 6 months compared to the typical child of 2 years old. The estimate of α_4 will determine the preference of the mother for investing in their child so that they reach the reference point or go beyond it.

The preference for consumption, α_1 is fixed to 1 for identification. All parameters are statistically significant. The preference for leisure, α_2 is positive for all reference points. The preference for the child's future skill, α_3 is also positive. However, the estimates for α_4 are all negative but smaller than α_3 in absolute value. This means that parents have a strong incentive to invest in their child up to the reference point, but beyond that their preferences are weaker. This highlights that parents' preferences for being above the reference point are stronger when the reference point is lower, that is, they are more concerned about their child being in a situation of severe developmental delay.[35]

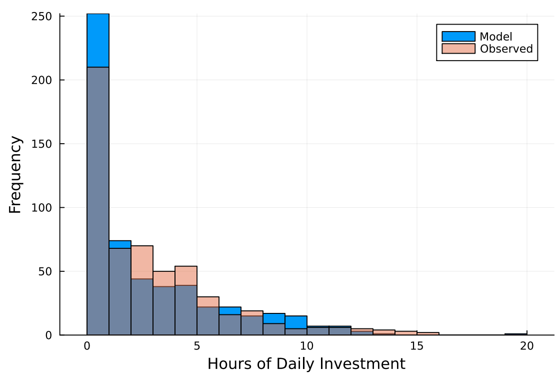
Figure 7 shows the distribution of daily investment hours that is implied by the model under the estimated parameters and compares it to the observed investment. In general, the estimated parameters using any references θ_{ref} have a good fit, but the model overestimates investments close to the bounds of possible investment hours, 0 and 20.

I now simulate a policy that increases the mean belief of the returns to investment, μ_{δ_2}. I simulate the policy for the different references points, and report the results in Table 14. This kind of simulation replicates the types of interventions that target parental beliefs. However, these interventions do not measure the uncertainty of the parents.

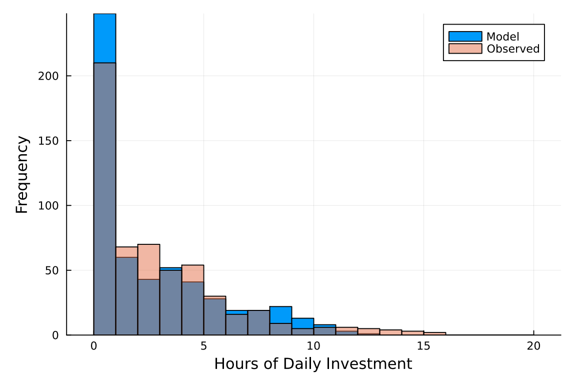
I first increase all individual beliefs about the returns to investment, μ_{i,δ_2}, by 10% for all individuals. I find that the increase in the mean belief leads to between 4% and 6% increase in the investment in children. The same is done for the uncertainty about returns to investment, σ^2_{i,δ_2}, and it also leads to an increase of about 3%, although with a decreasing trend as the reference points get relaxed.

[35] This is consistent with pilot interviews, where mothers were asked whether they valued having a child be developmentally advanced by age 2. While at the exploratory stage, mothers typically responded that they cared mostly about avoiding developmental delays, and did not have strong preferences about having a developmentally advanced child.

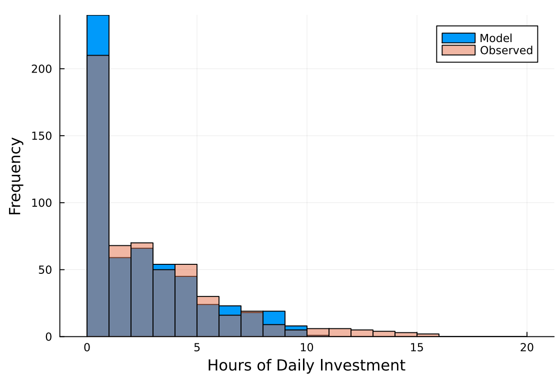
FIGURE 7. Model Predicted and Observed Hours of Investment



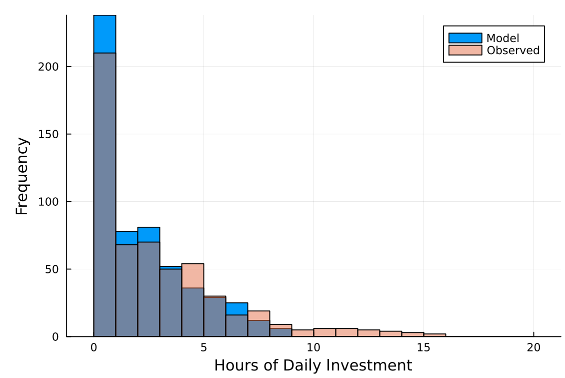
A. $\theta_{ref} = 18$



B. $\theta_{ref} = 20$



C. $\theta_{ref} = 22$



D. $\theta_{ref} = 24$

Note: This figure displays histograms of actual time investment, as reported by respondents, alongside predicted investment levels generated from the model using estimated preferences at various reference points.

This result contradicts the reduced form evidence presented in the previous section. In Table 2, I find that higher uncertainty is associated with lower investment, while the model predicts that increasing uncertainty leads to higher investment. Given that individuals with higher mean beliefs are also ones with lower uncertainty, it could be that the correlational regressions are simply reproducing the effect of mean beliefs.

TABLE 14. Percent Change in Daily Investment Hours Due to Change in Beliefs

	10% μ_{i,δ_2}	10% σ^2_{i,δ_2}
$\theta_{ref} = 18$	3.92	3.39
$\theta_{ref} = 20$	5.63	3.09
$\theta_{ref} = 22$	4.93	3.03
$\theta_{ref} = 24$	5.28	2.71

Note: This table shows the impact of increasing several belief variables on investment as predicted by the structural model and preference estimates for all four reference points. The first column shows the impact of an overall increase of 10% in individuals beliefs about the returns to investment. The second column shows the impact of an overall increase of 10% in individual uncertainty about the returns to investment.

The result that increasing uncertainty about returns to investment leads to an increase in investment may seem counterintuitive, particularly from the perspective of an asset investment problem, where higher uncertainty and risk aversion leads to lower investment overall, or a shift towards safe assets. While we cannot rule out that parents are risk-lovers with respect to child investment, it seems unlikely that this is the case.

The reason that we see this positive effect of uncertainty in this context is twofold. First, the estimates from model indicate that parents have a stronger incentive to invest in children when they believe that their child human capital is below the reference point. Second, most parents tend to have low mean beliefs about their child future human capital. By increasing the uncertainty about the returns to investment, parents now more "at-risk" of their child falling below specific developmental thresholds. This in turn leads them to invest more in their child.

This effect can be seen in Table 15, where I break down the 10% increase in σ^2_{i,δ_2}. The first column shows the percentage of individuals for which the increase in uncertainty leads to an increase or decrease in investment. About 64.5% observe an increase in investment, while 34% reduce their investments, with the remaining 1.5% not changing. The second column shows the average change in investment in hours. The third and fourth column shows the average mean beliefs of the individuals that increase their investment. Finally, the last column shows the average baseline investment of the individuals that increase their investment.

TABLE 15. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 18$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	64.46	0.05	0.05	0.04	0.96
Decrease in Investment	34.00	-0.01	0.17	0.10	3.95
No Change in Investment	1.54	0.00	0.26	0.15	12.15

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 18$. Results for other reference points can be found in the Appendix E. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.

On average, those who increase their investment due to the policy hold low beliefs about the returns to investment and to base human capital. This contrasts with those who decrease their investment, which tend to hold high beliefs about these parameters. Moreover, the same gap can be seen in baseline investment levels. The small fraction that does not change their investment are ones that already have large investment levels.

6. Conclusion

This paper develops a methodology that elicits both mean beliefs and belief uncertainty about the parameters of the skill production function in early childhood. The methodology and data collection procedure is motivated by a model of parental investment in children in which parents have do not have full knowledge of the skill production function. I elicit the belief distribution and allow for a flexible distribution of beliefs. I ask parents to report the youngest, most likely, and oldest ages a hypothetical child would learn to do some specific activities under different scenarios of initial skill and investment. I develop a simple estimation procedure that allows for correlation of measurement error across equations.

Additionally, I explore a new measure of parental subjective costs of investment in children. Recognizing that time investments represent opportunity costs for parents, I ask parents to report the amount of money they would be willing to pay to trade off investment and leisure. Together with the subjective belief distribution, I use this measure to estimate a model of parental investment in children which takes into account subjective beliefs.

I show that the collected data exhibit patterns that are consistent with the proposed theoretical model. Respondents report lower ages for the youngest, most likely, and oldest age of learning under the scenario of high investment and normal health, consistent with a model where children reach developmental milestones faster under more investment. They also report higher opportunity costs of investment under scenarios where they work more hours a day and have higher household income.

I estimate parental beliefs and find that parents in this sample have low mean beliefs about the returns to investment and uncertainty is relatively small. Nevertheless, individuals that have higher mean beliefs also tend to have lower subjective uncertainty. I also find that both mean beliefs and uncertainty correlate with actual time investment measures.

To illustrate the use of this data, I combine the literature on subjective beliefs and reference points and estimate a model of parental investment that incorporates these two aspects. I find that parents strongly value their child skill even when holding low mean beliefs. Moreover, they have a strong incentive to invest if their child is at risk of being at a developmental delay.

Counterfactual simulations show that an increase in uncertainty leads to an increase in investment. This seemingly counterintuitive result is due to the fact that those that increase their investment are ones that have low mean beliefs and low investment. The increase in uncertainty puts their child more "at-risk" of falling below developmental benchmarks, which induces an increase in investment.

In general, my findings indicate that belief heterogeneity in returns to investment is important in predicting actual time investment in children, and uncertainty about beliefs can be a relevant target for policy interventions. Moreover, the methodology developed in this paper is flexible enough to be transported to other contexts of child investments. Since I find that more pessimist parents tend to be more impacted by increases in uncertainty about their beliefs, it suggests that information interventions may have larger gains for less uncertain parents.

Several extensions and follow ups are possible. First, an overlooked aspect is the extent that risk aversion and intertemporal preferences impacts child investment. There are several method of elicitation of these features, but they are usually done in an abstract setting of money lotteries. If one wishes to study these aspects, it is necessary to develop context-specific elicitation procedures.

Second, eliciting the subjective cost of care presents several extension opportunities. The subjective cost of child care is poorly understood. The cost that women face in child care is multidimensional. Accurately estimating it is key to understanding why policies that subsidize child care, or ones that provide information interventions, do not translate into increases in investment. As Guryan, Hurst, and Kearney (2008) and Schoonbroodt (2018) point out, it is necessary to separate the costs of child care in at least two dimensions: (i) as a tradeoff against foregone earnings, and therefore, labor supply, and (ii) as a tradeoff against leisure and housework, usually "outside" working hours. In my elicitation method, I hold labor supply fixed and frame cost as a trade-off against leisure. A more flexible approach would be to elicit an additional cost framed as foregone earnings.

Finally, future research can conduct randomized controlled trials and use the methodology developed in this paper to explore the effects of information interventions on parental uncertainty, their subjective costs of care, and if they can mediate changes in actual investment in children.

References

Aizer, Anna, and Laura Stroud. 2010. "Education, Knowledge and the Evolution of Disparities in Health." Working Paper 15840, National Bureau of Economic Research.

Almås, Ingvild, Orazio Attanasio, and Pamela Jervis. 2023. "Economics and Measurement: New measures to model decision making." Working Paper 30839, National Bureau of Economic Research.

Arendt, Jacob Nielsen, Mads Lybech Christensen, and Anders Hjorth-Trolle. 2021. "Maternal education and child health: Causal evidence from Denmark." *Journal of Health Economics* 80: 102552.

Attanasio, Orazio, Teodora Boneva, and Christopher Rauh. 2019. "Parental Beliefs about Returns to Different Types of Investments in School Children." Working Paper 25513, National Bureau of Economic Research.

Attanasio, Orazio, Flávio Cunha, and Pamela Jervis. 2019. "Subjective Parental Beliefs. Their Measurement and Role." Working Paper 26516, National Bureau of Economic Research.

Attanasio, Orazio, and Katja Kaufmann. 2014. "Education choices and returns to schooling: Mothers' and youths' subjective expectations and their role by gender." *Journal of Development Economics* 109: 203–216.

Badev, Anton, and Flávio Cunha. 2012. "Learning about the Technology of Skill Formation." Working Paper, University of Pennsylvania.

Balestra, Piero, and Syoum Negassi. 1992. "A Random Coefficient Simultaneous Equation System with an Application to Direct Foreign Investment by French Firms." *Empirical Economics* 17: 205–220.

Basu, Arnab, and Ralitza Dimova. 2022. "Household Preferences and Child Labor in Rural Ethiopia." *Journal of African Economies*: forthcoming.

Biroli, Pietro, Teodora Boneva, Akash Raja, and Christopher Rauh. 2022. "Parental beliefs about returns to child health investments." *Journal of Econometrics* 231 (1): 33–57. Annals Issue: Subjective Expectations & Probabilities in Economics.

Bleemer, Zachary, and Basit Zafar. 2018. "Intended college attendance: Evidence from an experiment on college returns and costs." *Journal of Public Economics* 157: 184–211.

Boneva, Teodora, and Christopher Rauh. 2018. "Parental Beliefs about Returns to Educational Investments—The Later the Better?" *Journal of the European Economic Association* 16 (6): 1669–1711.

Bono, Emilia Del, Marco Francesconi, Yvonne Kelly, and Amanda Sacker. 2016. "Early Maternal Time Investment and Early Child Outcomes." *The Economic Journal* 126 (596): F96–F135.

Bruine de Bruin, Wändi, Alycia Chin, Jeff Dominitz, and Wilbert van der Klaauw. 2023. "Chapter 1 - Household surveys and probabilistic questions." In *Handbook of Economic Expectations*, edited by Rüdiger Bachmann, Giorgio Topa, and Wilbert van der Klaauw, 3–31: Academic Press.

Campbell, Frances A., Ramey , Craig T., Pungello , Elizabeth, Sparling , Joseph, , and Shari Miller-Johnson. 2002a. "Early Childhood Education: Young Adult Outcomes From the Abecedarian Project." *Applied Developmental Science* 6 (1): 42–57. Publisher: Routledge

_eprint: https://doi.org/10.1207/S1532480XADS0601_05.
Campbell, Frances A., Elizabeth P. Pungello, Margaret Burchinal, Kirsten Kainz, Yi Pan, Barbara H. Wasik, Oscar A. Barbarin, Joseph J. Sparling, and Craig T. Ramey. 2012. "Adult outcomes as a function of an early childhood educational program: An Abecedarian Project follow-up." *Developmental Psychology* 48 (4): 1033–1043. Place: US Publisher: American Psychological Association.
Campbell, Frances A., Craig T. Ramey, Elizabeth Pungello, Joseph Sparling, and Shari Miller-Johnson. 2002b. "Early childhood education: Young adult outcomes from the Abecedarian Project." *Applied Developmental Science* 6 (1): 42–57. Place: US Publisher: Lawrence Erlbaum.
Carneiro, Pedro, and Rita Ginja. 2016. "Partial Insurance and Investments in Children." *The Economic Journal* 126 (596): F66–F95.
Caucutt, Elizabeth, Lance Lochner, Joseph Mullins, and Youngmin Park. 2020. "Child skill production: Accounting for parental and market-based time and goods investments." Working Paper, NBER.
Cunha, Flavio, James J. Heckman, and Susanne M. Schennach. 2010. "Estimating The Technology of Cognitive and Noncognitive Skill Formation." *Econometrica* 78 (3): 883–931.
Cunha, Flávio. 2014. "Gaps in Early Investments in Children." Working Paper, Rice University.
Cunha, Flávio, Irma Elo, and Jennifer Culhane. 2013. "Eliciting Maternal Subjective Expectations about the Technology of Cognitive Skill Formation." WorkingPaper 19144, National Bureau of Economic Research.
Cunha, Flávio, Irma Elo, and Jennifer Culhane. 2022. "Maternal Subjective Expectations About the Technology of Skill Formation Predict Investments in Children One Year Later." *Journal of Econometrics* 231 (1): 3–32.
Currie, Janet, and Enrico Moretti. 2003. "Mother's Education and the Intergenerational Transmission of Human Capital: Evidence from College Openings." *The Quarterly Journal of Economics* 118 (4): 1495–1532.
Dahl, Gordon B., and Lance Lochner. 2012. "The Impact of Family Income on Child Achievement: Evidence from the Earned Income Tax Credit." *American Economic Review* 102 (5): 1927–1956.
Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. "Household Choices and Child Development." *The Review of Economic Studies* 81 (1): 137–185.
Delavande, Adeline. 2008. "Pill, Patch, or Shot? Subjective Expectations and Birth Control Choice." *International Economic Review* 49 (3): 999–1042.
Delavande, Adeline, Xavier Giné, and David McKenzie. 2011a. "Eliciting probabilistic expectations with visual aids in developing countries: how sensitive are answers to variations in elicitation design?" *Journal of Applied Econometrics* 26 (3): 479–497.
Delavande, Adeline, Xavier Giné, and David McKenzie. 2011b. "Measuring subjective expectations in developing countries: A critical review and new evidence." *Journal of Development Economics* 94 (2): 151–163.
Delavande, Adeline, and Basit Zafar. 2018. "University Choice: The Role of Expected Earnings, Non-pecuniary Outcomes and Financial Constraints." *Journal of Political Economy* 127 (5): 2343–2393.
Dizon-Ross, Rebecca. 2019. "Parents' Beliefs about Their Children's Academic Ability: Implica-

tions for Educational Investments." *American Economic Review* 109 (8): 2728–2765.
Duncan, Greg J., Kathleen M. Ziol-Guest, and Ariel Kalil. 2010. "Early-Childhood Poverty and Adult Attainment, Behavior, and Health." *Child Development* 81 (1): 306–325. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-8624.2009.01396.x.
Folbre, Nancy, Jayoung Yoon, Kade Finnoff, and Allison Sidle Fuligni. 2005. "By What Measure? Family Time Devoted to Children in the United States." *Demography* 42 (2): 373–390.
Garca, Jorge Luis, James J. Heckman, Duncan Ermini Leaf, and María José Prados. 2020. "Quantifying the Life-Cycle Benefits of an Influential Early-Childhood Program." *The journal of political economy* 128 (7): 2502–2541.
Giustinelli, Pamela. 2016. "Group Decision Making with Uncertain Outcomes: Unpacking Child–Parent Choice of the High School Track." *International Economic Review* 57 (2): 573–602.
Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. "Parental Education and Parental Time with Children." *Journal of Economic Perspectives* 22 (3): 23–46.
Heckman, James J., and Ganesh Karapakula. 2019. "Intergenerational and Intragenerational Externalities of the Perry Preschool Project."
Heckman, James, Seong Hyeok Moon, Rodrigo Pinto, Peter Savelyev, and Adam Yavitz. 2010. "Analyzing Social Experiments as Implemented: A Teexamination of the Evidence From the HighScope Perry Preschool Program." *Quantitative Economics* 1 (1): 1–46.
Jensen, Robert. 2010. "The (Perceived) Returns to Education and the Demand for Schooling." *The Quarterly Journal of Economics* 125 (2): 515–548.
Judge, George G., William E. Griffiths, R. Carter Hill, Helmut Lütkepohl, and Tsoung-Chao Lee. 1988. *The Theory and Practice of Econometrics*. 2nd ed. New York: Wiley.
Kinsler, Josh, and Ronni Pavan. 2021. "Local Distortions in Parental Beliefs over Child Skill." *Journal of Political Economy* 129 (1): 81–100. Publisher: The University of Chicago Press.
Korfmacher, Jon, Beth Green, Fredi Staerkel, Carla Peterson, Gina Cook, Lori Roggman, Richard A. Faldowski, and Rachel Schiffman. 2008. "Parent Involvement in Early Childhood Home Visiting." *Child & Youth Care Forum* 37 (4): 171–196.
List, John A., Julie Pernaudet, and Dana L. Suskind. 2021. "Shifting Parental Beliefs About Child Development to Foster Parental Investments and Improve School Readiness Outcomes." *Nature Communications* 12 (1): 5765.
List, John A., Paramita Sinha, and Michael H. Taylor. 2006. "Using Choice Experiments to Value Non-Market Goods and Services: Evidence from Field Experiments." *The B.E. Journal of Economic Analysis & Policy* 6 (2).
Ma, Xin, Jianping Shen, Huilan Y. Krenn, Shanshan Hu, and Jing Yuan. 2016. "A Meta-Analysis of the Relationship Between Learning Outcomes and Parental Involvement During Early Childhood Education and Early Elementary Education." *Educational Psychology Review* 28 (4): 771–801.
Manski, Charles F. 1993. "Adolescent Econometricians: How Do Youth Infer the Returns to Schooling?" In *Studies of Supply and Demand in Higher Education,* 43–60: University of Chicago Press.
Manski, Charles F. 2004. "Measuring Expectations." *Econometrica* 72 (5): 1329–1376.
Mira, Pedro. 2007. "Uncertain Infant Mortality, Learning, and Life-cycle Fertility." *International*

Economic Review 48 (3): 809–846.

Patnaik, Arpita, Joanna Venator, Matthew Wiswall, and Basit Zafar. 2022. “The Role of Heterogeneous Risk Preferences, Discount Rates, and Earnings Expectations in College Major Choice.” *Journal of Econometrics* 231 (1): 98–122.

Schoonbroodt, Alice. 2018. “Parental child care during and outside of typical work hours.” *Review of Economics of the Household* 16 (2): 453–476.

Schweinhart, Lawrence J., John R. Berrueta-Clement, W. Steven Barnett, Ann S. Epstein, and David P. Weikart. 1985. “Effects of the Perry Preschool Program on Youths Through Age 19: A Summary.” *Topics in Early Childhood Special Education* 5 (2): 26–35. Publisher: SAGE Publications Inc.

Sovero, Veronica. 2018. “Risk Preferences and Child Investments: Evidence From Mexico.” *Review of Economics of the Household* 16 (4): 1027–1047.

Stantcheva, Stefanie. 2023. “How to Run Surveys: A Guide to Creating Your Own Identifying Variation and Revealing the Invisible.” *Annual Review of Economics* 15 (Volume 15, 2023): 205–234.

Stinebrickner, Ralph, and Todd Stinebrickner. 2014. “Academic Performance and College Dropout: Using Longitudinal Expectations Data to Estimate a Learning Model.” *Journal of Labor Economics* 32 (3): 601–644.

Swamy, P. A. V. B. 1970. “Efficient Inference in a Random Coefficient Regression Model.” *Econometrica* 38 (2): 311–323.

Tabetando, Rayner. 2019. “Parental risk aversion and educational investment: panel evidence from rural Uganda.” *Review of Economics of the Household* 17 (2): 647–670.

Tanaka, Yuki, and Takashi Yamano. 2015. “Risk and Time Preference on Schooling: Experimental Evidence from a Low-Income Country.” Working Paper 14-24, National Graduate Institute for Policy Studies.

Wang, Fan, Esteban Puentes, Jere R. Behrman, and Flávio Cunha. 2022. “You are what your parents expect: Height and local reference points.” *Journal of Econometrics*: 105269.

Wiswall, Matthew, and Basit Zafar. 2015. “Determinants of College Major Choice: Identification using an Information Experiment.” *The Review of Economic Studies* 82 (2): 791–824.

Zafar, Basit. 2013. “College Major Choice and the Gender Gap.” *Journal of Human Resources* 48 (3): 545–595.

Appendix A. Survey Instrument

A.1. Belief Elicitation - Instructions

Throughout this section, we will refer to different scenarios of health of the baby and hours of interaction the mother spends with the baby.

1) A "normal health" baby is one whose gestation lasted 9 months, weighed 8 pounds, and measured 20 inches at birth. A "poor health" baby is one whose gestation lasted 7 months, weighed 5 pounds, and measured 18 inches at birth.

2) A "high intensity" interaction is one in which the mothers spends 6 hours a day with the baby in active interaction, while a "low intensity" one the mother spends 2 hours a day with the baby in active interaction. These interactions includes activities such as:

- soothing the baby when he/she is upset;
- moving the baby's arms and legs around playfully;
- playing peek-a-boo with the baby;
- singing songs with the baby;
- speaking to the baby;
- feeding, nursing, bathing, attending to health needs;

We would like you to consider a hypothetical scenario involving a mother and her baby. In this scenario, the baby's health can either be good or poor, and the mother's interaction with the baby can be either high intensity or low intensity. After considering these factors, we would like you to determine the youngest, most likely, and oldest age (in months) at which the baby in this specific situation will learn to perform a certain activity.

To illustrate, let us consider the example of a baby with "normal health" and a "low intensity" interaction between mother and baby. In this case, we would like you to provide your personal belief on the youngest, most likely, and oldest ages at which this baby will learn to walk at least 5 steps by itself. To help you understand the youngest, most likely, and oldest age, we suggest imagining 10 identical babies, with some learning to perform the activity at different ages. In this way, the youngest age would be the earliest at which any of the babies learn the activity, the most likely age would be the age at which most of the babies learn the activity, and the oldest age would be the latest at which any of the babies learn the activity.

In total, there will be 16 questions of this nature, each pertaining to a different scenario and activity. While there are no right or wrong answers, we ask that you carefully consider each situation and activity before giving us your honest personal belief.

A.2. Stated Choice - Instructions

In this section, we will refer to the active interaction time a mother spends with her baby as hours of mother-child interaction. Here are some examples of activities a mom does during active time interactions:

- soothing the baby when he/she is upset;
- moving the baby's arms and legs around playfully;
- playing peek-a-boo with the baby;
- singing songs with the baby;
- speaking to the baby;
- feeding, nursing, bathing, attending to health needs;

What is important to highlight is that active interaction time is one where the main and sole focus of the mother is in the baby.

When the mother is at home but not in active interaction time, we call this leisure or passive interaction time. The mother can still be together with the baby, but the baby is not the main focus of the activity. Here are some examples of activities a mom does during passive time interactions:

- Grocery shopping with baby;
- Browsing social media apps on smartphone with baby at your side;
- Nap time for baby;
- Household chores (cleaning, cooking, etc) while baby is at your side;
- Exercising at home or at the gym;

- Watching TV;

In all these activities, although the mother may check on the baby every 5-10 minutes, she is not exclusively focused on the baby during these activities.

We will also refer to the health of the baby.

A “normal health” baby is one whose gestation lasted 9 months, the baby weighed 8 pounds and measured 20 inches at birth. A “poor health” baby is one whose gestation lasted 7 months, the baby weighed 5 pounds and measured 18 inches at birth.

In this section of the survey, we will ask you to imagine yourself in a new household, composed of you, a partner, and a hypothetical baby (that is, not one of your current children). We will present different situations of household income and health of the baby.

Then, we will ask you to imagine a situation where you want to spend 1 hour away from your baby after work every day for one month (not on weekends). For example, you may want to set a time for your personal rest, or you want to practice a hobby. You ask a friend to look after your baby for that hour. Your friend is a careful person who will ensure that your baby is well taken care of, but they will not engage in active interaction with your baby.

We will ask you to choose the highest hourly rate you would be willing to pay your friend during weekdays for one month under different working hour situations. We assume that there are 20 weekdays in the month. If you would not be willing to stay 1 hour away from the baby, please select $0 dollars.

For example, suppose that when you work 8 hours a day every day, the highest rate you would pay your friend is $10. This is equal to 1 hour times 20 weekdays in a month times $10/hour you are willing to pay, which is equal to $200 per month. The answer would look like this:

We know these questions are not easy to answer. Note that there is no right or wrong, good or bad, answer, we are just interested in what you personally think. Please try to consider each scenario carefully and tell us what you personally believe is the best option. We ask that you make an effort to thoughtfully answer all questions.

Appendix B. Data Details

This appendix provides additional details on the data cleaning procedures used in the analysis.

The survey was completed by 723 women between the ages of 18 and 40, all of whom had at least one child, with the oldest child no older than five years old. Qualtrics, the platform contracted to administer the online survey, aimed to match respondents to U.S. Census benchmarks for education and household income. However, due to limitations in their available panel and challenges in fulfilling the target quotas, the final sample ended up skewing toward individuals with higher educational attainment and household income.

The survey design did not require responses to any questions beyond the consent form and initial screening. Participants were free to skip any subsequent question. Following the methodology in Stantcheva (2023), we included a question on self-reported honesty. Twelve individuals indicated that they had not answered the survey truthfully and were therefore excluded from the sample. We further dropped 91 individuals who failed to provide responses to all required questions, leaving a total of 620 respondents.

Next, we identified cases of non-substantive responses, where individuals gave mechanically repetitive or non-informative answers, which we interpret as non-response behavior. For instance, in the belief elicitation questions asking for youngest, median, and oldest ages for milestone completion, some respondents entered values such as 0, 0, 0 or 20, 20, 20. Similarly, in the price sensitivity instrument, some individuals reported a value of zero across all scenarios. We also drop individuals that did not answer or skipped at least one scenario. Based on these criteria, we excluded 31 individuals due to uniform responses in the belief module and 82 individuals for the same issue in the price instrument, for a total of 113 additional removals. This step reduced the working sample to 507 respondents.

Finally, for model estimation, we applied additional consistency checks. We dropped individuals whose reported time allocations exceeded the feasible upper bound of 16 hours per day for either labor supply or child investment. We also calculated available household income using respondents' answers in the price module and excluded those with implied negative disposable income. These constraints led to the removal of 54 more respondents, resulting in a final sample of 453 individuals used in the structural estimation.

Appendix C. Seemingly Unrelated Random Coefficients Estimator

Let N denote the number of individuals, T the number of observations for each individual, and L the number of parameters to be estimated. We would like to estimate the following system:

$$y_i = Z_i\beta + (\varepsilon_i + Z_i\eta_i) = Z_i\beta + u_i,$$

First, for each $i = 1, \ldots, N$, run an OLS regression to prepare for the FGLS Seemingly Unrelated regression (SUR) estimator:

$$\begin{aligned}
\hat{b}_{0,i} &= (Z_i'Z_i)^{-1}(Z_i'y_i),\\
\hat{\Omega}_{0,i} &= \hat{u}_{0,i}'\hat{u}_{0,i}/(T-L),\\
\hat{V}_{0,i} &= (Z_i'(\hat{\Omega}_{0,i}^{-1} \otimes I_T)Z_i)^{-1}.
\end{aligned}$$

Then, compute the feasible SUR estimates for each individual:

$$\begin{aligned}
\hat{b}_{1,i} &= (Z_i'(\hat{\Omega}_{0,i}^{-1} \otimes I_T)Z_i)^{-1}(Z_i'(\hat{\Omega}_{0,i}^{-1} \otimes I_T)\,y_i),\\
\hat{\Omega}_{1,i} &= \hat{u}_{1,i}'\hat{u}_{1,i}/(T-L),\\
\hat{V}_{1,i} &= (Z_i'(\hat{\Omega}_{1,i}^{-1} \otimes I_T)Z_i)^{-1}.
\end{aligned}$$

In the second step, we obtain the feasible estimates for the covariance matrices of the random coefficients and measurement error.[36] Denote $\bar{\hat{b}}_1 = \frac{1}{N}\sum_{i=1}^{N}\hat{b}_{1,i}$:

$$\begin{aligned}
\hat{\Delta} &= \frac{1}{N-1}\sum_{i=1}^{N}(\hat{b}_{1,i}\hat{b}_{1,i}') - N\bar{\hat{b}}_1\bar{\hat{b}}_1',\\
\hat{\Pi}_i &= Z_i\hat{\Delta}Z_i' + (I_T \otimes \hat{\Omega}_{1,i}),\\
\hat{V}_{1,i} &= (Z_i'(\hat{\Omega}_{1,i}^{-1} \otimes I_T)Z_i)^{-1}.
\end{aligned}$$

Then, the efficient SUR estimator is:

$$\begin{aligned}
\hat{\beta} &= (\sum_{i=1}^{N} Z_i'(\hat{\Pi})^{-1}Z_i)^{-1}(\sum_{i=1}^{N} Z_i'(\hat{\Pi})^{-1}\,y_i),\\
Var(\hat{\beta}) &= \frac{1}{N}\sum_{i=1}^{N}(\hat{\Delta} + \hat{V}_{1,i})^{-1}.
\end{aligned}$$

[36]Technically, the estimator for $\hat{\Delta}$ also includes the term $-\frac{1}{N}\sum_{i=1}^{N} V_i$. However, as pointed out by Swamy (1970), this often leads to a computationally nonpositive definite matrix, and this latter term is negligible in large samples.

To obtain an efficient estimator of the individual level paramater β_i, I adapt the estimator for Swamy (1970) found in Judge et al. (1988). Denote $A_i = (\hat{\Delta}^{-1} + \hat{V}_i^{-1})^{-1}\hat{\Delta}^{-1}$. Then:

$$\hat{\beta}_i = (\hat{\Delta}^{-1} + \hat{V}_i^{-1})^{-1}(\hat{\Delta}^{-1}\hat{\beta} + \hat{V}_i^{-1}\hat{b}_{1,i}),$$
$$Var(\hat{\beta}_i) = Var(\hat{\beta}) + (I_L - A_i)(\hat{V}_i^{-1} - Var(\hat{\beta}))(I_L - A_i)'.$$

Appendix D. Robustness of Distributional Assumptions

In this section I present estimates for the parameters of the subjective production function using different distributional assumptions for respondent's answers. The main results of the paper assumes that respondent's answers follow a Triangular distribution, where the oldest and youngest ages reported are the extremes of the triangular distribution, and the most likely answer corresponds to the mode, which is the peak of the triangle.

This kind of elicitation procedure is also known as "three-point estimation." One commonly used distribution is the PERT distribution, which is a transformation of the more general Beta distribution. In practice, it resembles a smoothed triangular distribution. From the minimum a, most likely, b, and maximum values c, the mean of this distribution is given by $\mu = \frac{a+4b+c}{6}$, its median by $Med = \frac{a+6b+c}{8}$, and variance by $\frac{(\mu-a)(c-\mu)}{7}$.

Delavande, Giné, and McKenzie (2011b) and Delavande, Giné, and McKenzie (2011a) show that in the context of some developing countries (namely, India and Tonga), respondents do not interpret minimum and maximum questions as representing 0 and 100 chance probabilities of the cumulative distribution function. They show that most respondents interpret these bounds as representing anywhere between the 90th and 95th percentiles for the maximum. As such, I also estimate the parameters but instead assuming that the youngest and oldest answers represent either the 1st and 99th, 5th and 95th, and 10th and 90th percentiles. This involves solving a simple system of equations I solve $a = F(x_{n\text{–th}}; a, b, c)$; $c = F(x_{(100-n)\text{–th}}; a, b, c)$, where $x_{(n\text{–th}}$ and $x_{(100-n)\text{–th}}$ are the percentile values I want to find and $F(; a, b, c)$ is the cdf of the triangular distribution with parameters a, b, c.

Table A1 presents the results for these different assumption, including the baseline assumption of a triangular distribution. The parameters of the mean subjective production function are stable for all distributions. However, the variance estimates are somewhat sensitive to distributional assumptions, particularly the ones that extend the distribution beyond the youngest and oldest values. This is not surprising, since the outcome variable $Var(\ln\theta_{i,1}|\Omega_i)$ is based off the IQR of the distribution of answers. Therefore, the distributions that extend participant answers also inflate IQR estimates. The estimates for $\sigma^2_{\delta_2}$ using the 10th and 90th percentiles are 3 times larger than the baseline assumption.

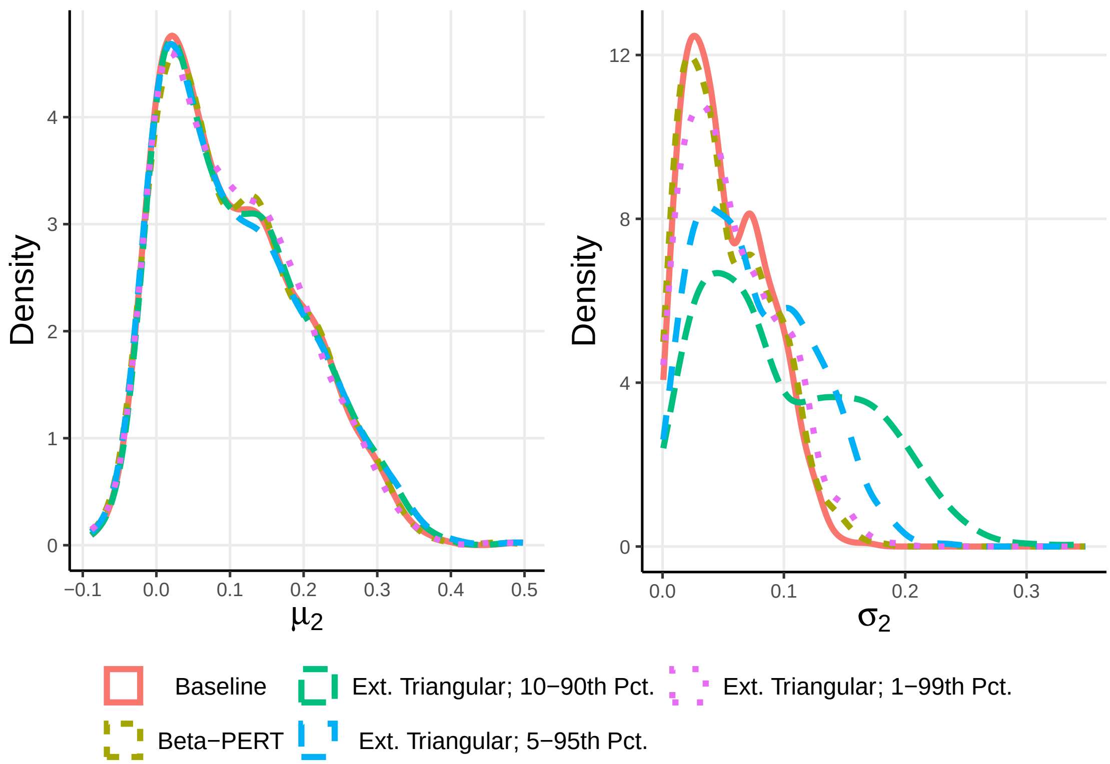
This discrepancy can be seen in Figure A1. The distribution of μ_{i,δ_2} does not vary much by assumption, while the distribution of σ_{i,δ_2} becomes more spread out the larger the extension of the extreme values.

TABLE A1. Estimates of Mean Subjective Production Function Parameters - Robustness

	Baseline	PERT	Triag. 1–99th	Triag. 5–95th	Triag. 10–90th
μ_{δ_0}	2.897***	2.892***	2.889***	2.893***	2.891***
	(0.0285)	(0.0286)	(0.0282)	(0.0291)	(0.0286)
μ_{δ_1}	0.064***	0.063***	0.064***	0.064***	0.064***
	(0.0045)	(0.0046)	(0.0045)	(0.0046)	(0.0045)
μ_{δ_2}	0.103***	0.103***	0.102***	0.105***	0.106***
	(0.0084)	(0.0088)	(0.0086)	(0.0087)	(0.0085)
σ^2_0	0.031***	0.031***	0.030***	0.063***	0.090***
	(0.0023)	(0.0023)	(0.0027)	(0.0045)	(0.0063)
$\sigma^2_{\delta_1}$	0.001**	0.001**	0.001**	0.002***	0.004***
	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0006)
$\sigma^2_{\delta_2}$	0.004***	0.004***	0.004***	0.007***	0.013***
	(0.0006)	(0.0007)	(0.0008)	(0.0011)	(0.0020)
$\sigma_{\delta_1,\delta_2}$	−0.002**	−0.002**	−0.003***	−0.004***	−0.007***
	(0.0006)	(0.0007)	(0.0008)	(0.0011)	(0.0019)

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows aggregate estimates of equations (8) and (9) with different distributional assumptions about answers from youngest, most likely, and oldest ages. All distributions use the most likely answer as the mode. Baseline and PERT uses the youngest and oldest as the extremes of a triangular distribution. The last three columns assumes that the youngest and oldest matches specific percentiles of a triangular distribution.

FIGURE A1. Distribution of Estimates of μ_{i,δ_2} and σ_{i,δ_2}



Note: This figures shows the distribution of estimates of μ_{i,δ_2} and σ_{i,δ_2} with different distributional assumptions about answers from youngest, most likely, and oldest ages. All distributions use the most likely answer as the mode. Baseline and PERT uses the youngest and oldest as the extremes of a triangular distribution. The last three columns assumes that the youngest and oldest matches specific percentiles of a triangular distribution.

Appendix E. Additional Tables and Figures

FIGURE A2. Distribution of Ages by Scenario for "Count 3 Objects Correctly"

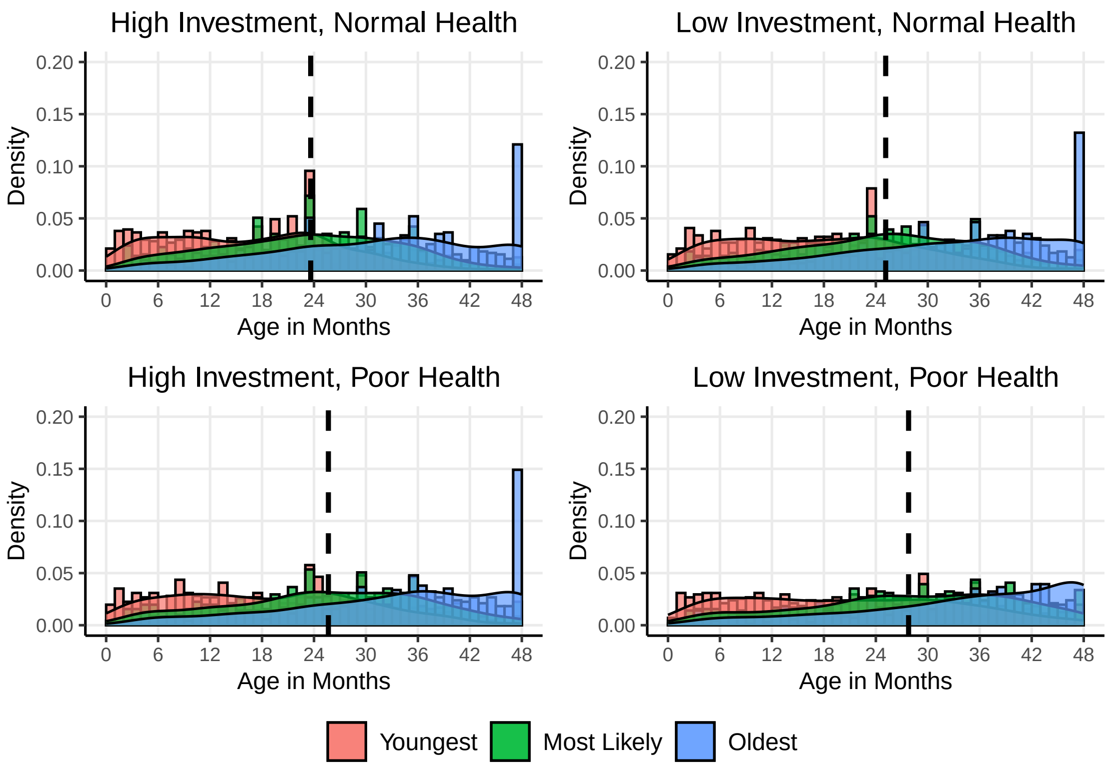


FIGURE A3. Distribution of Ages by Scenario for "Say First and Last Name Together"

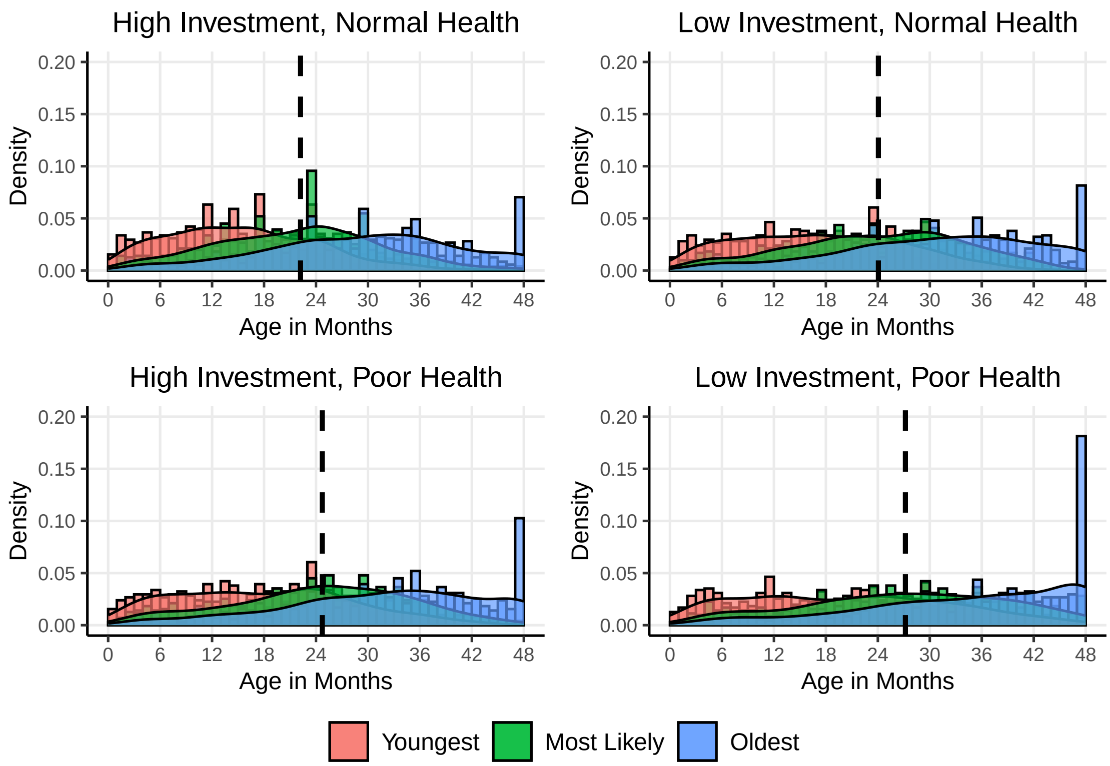


FIGURE A4. Distribution of Ages by Scenario for "Know Own Age and Sex"

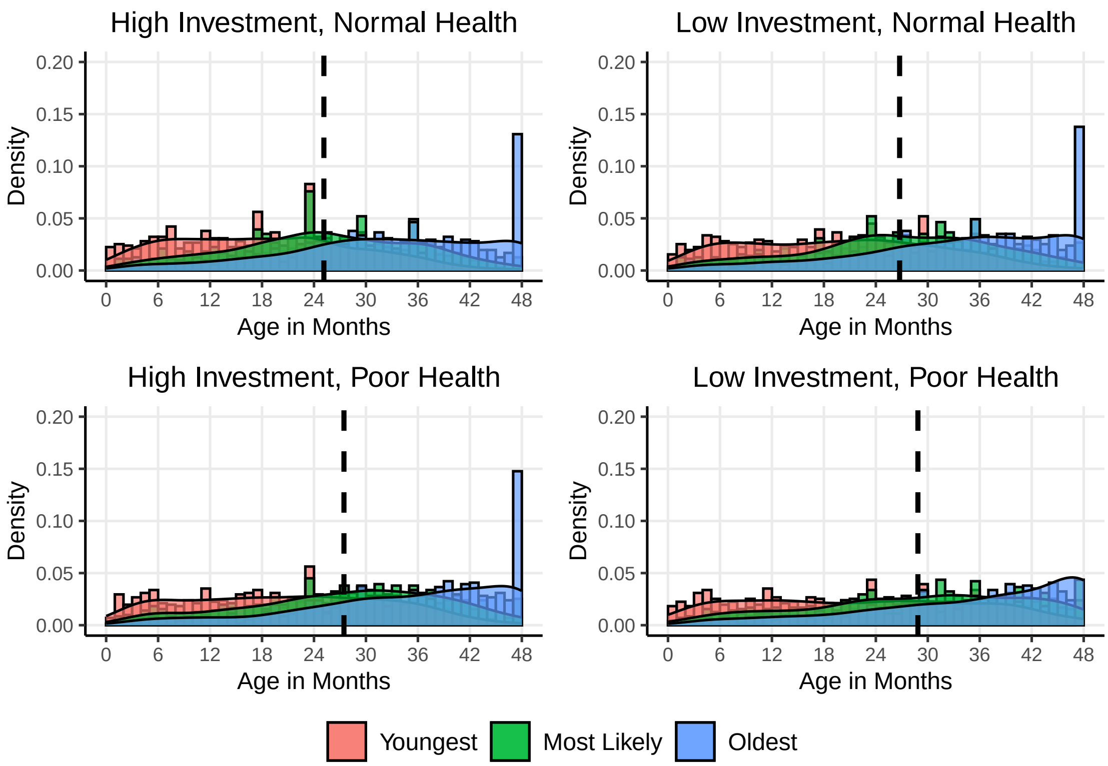


TABLE A2. Descriptive Statistics Investment

	Good Health, Household Income of $2,000					
	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	12.40	6.72	11.10	6.89	8.14	7.51
	Good Health, Household Income of $4,000					
	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	14.15	6.51	12.29	6.56	9.32	7.91
	Good Health, Household Income of $6,000					
	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	15.28	7.24	13.30	6.90	10.55	8.16
	Poor Health, Household Income of $2,000					
	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	12.42	8.14	11.24	7.73	8.87	8.39
	Poor Health, Household Income of $4,000					
	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	13.40	8.00	12.18	7.63	9.51	8.31
	Poor Health, Household Income of $6,000					
	Work 8 Hours		Work 4 Hours		Work 0 Hours	
	Mean	SD	Mean	SD	Mean	SD
Willingness to Pay ($)	14.92	8.74	13.47	8.12	11.03	8.98

This table shows

TABLE A3. Estimates of Willingness to Pay System

Panel B: Aggregate Estimates	
p_i	12.238***
	(0.225)
Work Hours	1.663***
	(0.108)
Income	1.007***
	(0.079)
θ_0	−0.039
	(0.101)
Panel B: Individual Estimates	
Parameter	% Significant
p_i	99.34%

Note: Robust standard errors in parenthesis. *p<0.1; **p<0.05; ***p<0.01. This table shows aggregate estimates of the willingness to pay equations 10 and the individual-level predicted estimates of p_i. The percentage of significant estimates is calculated using a 10% significance level, and the null hypothesis is that the coefficient is equal to zero.

TABLE A4. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref})$ = 20

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	62.25	0.05	0.05	0.03	0.96
Decrease in Investment	35.32	0.00	0.17	0.10	3.34
No Change in Investment	2.43	0.00	0.27	0.15	11.61

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref})$ = 20. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.

TABLE A5. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 22$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	62.25	0.05	0.05	0.04	0.83
Decrease in Investment	34.66	0.00	0.16	0.10	3.31
No Change in Investment	3.09	0.00	0.25	0.15	10.78

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 22$. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.

TABLE A6. Breakdown of Change in Investment Due to a 10% Increase in σ^2_{i,δ_2} - $log(\theta_{ref}) = 24$

	% by Sign	Nominal Change (Hours)	μ_2	μ_1	Baseline $\hat{x}$
Increase in Investment	60.49	0.05	0.05	0.04	0.88
Decrease in Investment	34.88	0.00	0.16	0.09	3.27
No Change in Investment	4.64	0.00	0.22	0.13	10.55

Note: This table breaks down the impact of increasing belief uncertainty about the returns to investment by 10% on investment as predicted by the structural model and preference estimates for $log(\theta_{ref}) = 24$. Each line shows baseline investment, mean beliefs, and the nominal change in hours conditional on whether the increase in uncertainty leads to an increase, decrease, or no change in investment.